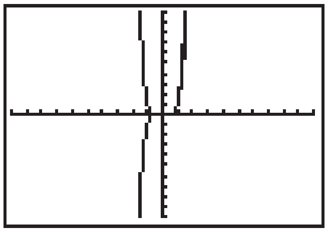
Chapter 11 Exponential and Logarithmic Function

11-1 Real Exponents

Page 695 Graphing Calculator Exploration

1.



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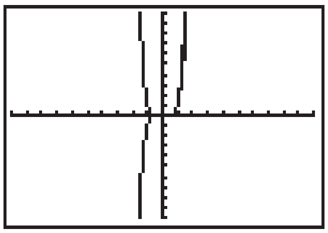


3. $a^m \cdot a^n = a^{m+n}$

4. $(a^m)^n = a^{mn}$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, when $b \neq 0$

Page 700 Check for Understanding

1. The quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.

2. If the base were negative and the denominator were even, then we would be taking an even root of a negative number, which is undefined as a real number.

3. Laura is correct. The negative exponent of 10 represents a fraction with a numerator of 1 and a denominator of a positive power of 10. The product of this fraction and a number between 1 and 10 is between 0 and 1.

4.
$$\begin{aligned} 5^{-4} &= \frac{1}{5^4} \\ &= \frac{1}{625} \end{aligned}$$

5.
$$\begin{aligned} \left(\frac{9}{16}\right)^{-2} &= \frac{1}{\left(\frac{9}{16}\right)^2} \\ &= \left(\frac{16}{9}\right)^2 \\ &= \frac{16^2}{9^2} \\ &= \frac{256}{81} \end{aligned}$$

6.
$$\begin{aligned} 216^{\frac{1}{3}} &= (6^3)^{\frac{1}{3}} \\ &= 6^{\frac{3}{3}} \\ &= 6 \end{aligned}$$

7.
$$\begin{aligned} \sqrt{27} \cdot \sqrt{3} &= 27^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \\ &= (3^3)^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \\ &= 3^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} \\ &= 3^{\frac{4}{2}} \\ &= 3^2 \\ &= 9 \end{aligned}$$

8.
$$\begin{aligned} 32^{\frac{3}{5}} &= (2^5)^{\frac{3}{5}} \\ &= 2^{\frac{15}{5}} \\ &= 2^3 \\ &= 8 \end{aligned}$$

9.
$$\begin{aligned} (3a^{-2})^3 \cdot 3a^5 &= 3^3 \cdot a^{-6} \cdot 3a^5 \\ &= 3^4 a^{-1} \\ &= 81a^{-1} \text{ or } \frac{81}{a} \end{aligned}$$

10.
$$\begin{aligned} \sqrt{m^3n^2} \cdot \sqrt{m^4n^5} &= (m^3n^2)^{\frac{1}{2}} \cdot (m^4n^5)^{\frac{1}{2}} \\ &= m^{\frac{3}{2}}n^{\frac{2}{2}} \cdot m^{\frac{4}{2}}n^{\frac{5}{2}} \\ &= m^{\frac{7}{2}}n^{\frac{7}{2}} \text{ or } m^3n^3\sqrt{mn} \end{aligned}$$

11.
$$\begin{aligned} \sqrt{\frac{8^n \cdot 2^7}{4^{-n}}} &= \left(\frac{8^n \cdot 2^7}{4^{-n}}\right)^{\frac{1}{2}} \\ &= \frac{8^{\frac{n}{2}} \cdot 2^{\frac{7}{2}}}{4^{-\frac{n}{2}}} \\ &= \frac{(2^3)^{\frac{n}{2}} \cdot 2^{\frac{7}{2}}}{(2^2)^{-\frac{n}{2}}} \\ &= \frac{2^{\frac{3n}{2}} \cdot 2^{\frac{7}{2}}}{2^{-\frac{2n}{2}}} \\ &= 2^{\frac{3n}{2}} \cdot 2^{\frac{7}{2}} \cdot 2^{\frac{2n}{2}} \\ &= 2^{\frac{5n+7}{2}} \\ &= 2^{2n+3} \cdot 2^{\frac{n+1}{2}} \\ &= 2^{2n+3}\sqrt{2^{n+1}} \end{aligned}$$

12.
$$\begin{aligned} (2x^4y^8)^{\frac{1}{2}} &= 2^{\frac{1}{2}} \cdot (x^4)^{\frac{1}{2}} \cdot (y^8)^{\frac{1}{2}} \\ &= 2^{\frac{1}{2}} \cdot x^{\frac{4}{2}} \cdot y^{\frac{8}{2}} \\ &= 2^{\frac{1}{2}} \cdot x^2y^4 \quad \text{or} \quad x^22^4\sqrt{2} \end{aligned}$$

13.
$$\begin{aligned} \sqrt{169x^5} &= (169x^5)^{\frac{1}{2}} \\ &= 169^{\frac{1}{2}} \cdot (x^5)^{\frac{1}{2}} \\ &= 13x^{\frac{5}{2}} \end{aligned}$$

14.
$$\begin{aligned} \sqrt[4]{a^2b^3c^4d^5} &= (a^2b^3c^4d^5)^{\frac{1}{4}} \\ &= (a^2)^{\frac{1}{4}}(b^3)^{\frac{1}{4}}(c^4)^{\frac{1}{4}}\,(d^5)^{\frac{1}{4}} \\ &= |a|^{\frac{1}{2}}\,|b|^{\frac{3}{4}}\,|c|\,\,|d|^{\frac{5}{4}} \end{aligned}$$

15.
$$\begin{aligned} 6^{\frac{1}{4}}b^{\frac{3}{4}}c^{\frac{1}{4}} &= (6b^3c)^{\frac{1}{4}} \\ &= \sqrt[4]{6b^3c} \end{aligned}$$

16.
$$\begin{aligned} 15x^{\frac{1}{3}}y^{\frac{1}{5}} &= 15x^{\frac{5}{15}}y^{\frac{3}{15}} \\ &= 15\,(x^5y^3)^{\frac{1}{15}} \\ &= 15\sqrt[15]{x^5y^3} \end{aligned}$$

17.
$$\begin{aligned} \sqrt[3]{p^4q^6r^5} &= (p^4q^6r^5)^{\frac{1}{3}} \\ &= (p^4)^{\frac{1}{3}}\,(q^6)^{\frac{1}{3}}\,(r^5)^{\frac{1}{3}} \\ &= p^{\frac{4}{3}}q^2r^{\frac{5}{3}} \\ &= pq^2r\sqrt[3]{pr^2} \end{aligned}$$

18.
$$\begin{aligned} y^{\frac{4}{5}} &= 34 \\ \left(y^{\frac{4}{5}}\right)^{\frac{5}{4}} &= 34^{\frac{5}{4}} \\ y &= (34^5)^{\frac{1}{4}} \\ y &\approx 82.1 \end{aligned}$$

19. $A = \pi r^2 \qquad r = 3.875 \times 10^{-7}$ m
$A = \pi(3.875 \times 10^{-7}\text{ m})^2$
$= \pi(3.875)^2 \times (10^{-7})^2\text{ m}^2$
$= \pi(15.015625 \times 10^{-14})\text{ m}^2$
$\approx 4.717 \times 10^{-13}\text{ m}^2$

Pages 700–703 Exercises

20. $(-6)^{-4} = \frac{1}{(-6)^4}$
$= \frac{1}{1296}$

21. $-6^{-4} = -\left(\frac{1}{6^4}\right)$
$= -\frac{1}{1296}$

22. $(5 \cdot 3)^2 = 15^2$
$= 225$

23. $\frac{2^4}{2^{-1}} = 2^{4-(-1)}$
$= 2^5$
$= 32$

24. $\left(\frac{7}{8}\right)^{-3} = \frac{1}{\left(\frac{7}{8}\right)^3}$
$= \left(\frac{8}{7}\right)^3$
$= \frac{8^3}{7^3}$
$= \frac{512}{343}$

25. $(3^{-1} + 3^{-3})^{-1} = \frac{1}{3^{-1} + 3^{-1}}$
$= \frac{1}{\frac{1}{3} + \frac{1}{9}}$
$= \frac{1}{\frac{4}{9}}$
$= \frac{9}{4}$

26. $81^{\frac{1}{2}} = (9^2)^{\frac{1}{2}}$
$= 9$

27. $729^{\frac{1}{3}} = (9^3)^{\frac{1}{3}}$
$= 9$

28. $\frac{27}{27^{\frac{2}{3}}} = \frac{3^3}{(3^3)^{\frac{2}{3}}}$
$= \frac{3^3}{3^2}$
$= 3$

29. $2^{\frac{1}{2}} \cdot 12^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot (2 \cdot 6)^{\frac{1}{2}}$
$= 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 6^{\frac{1}{2}}$
$= 2 \cdot 6^{\frac{1}{2}}$
$= 2\sqrt{6}$

30. $64^{\frac{1}{2}} = (2^6)^{\frac{1}{12}}$
$= 2^{\frac{1}{2}}$ or $\sqrt{2}$

31. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$
$= \frac{1}{(2^4)^{\frac{1}{4}}}$
$= \frac{1}{2}$

32. $\frac{(3^7)(9^4)}{\sqrt{27^6}} = \frac{3^7(3^2)^4}{(3^3)^{\frac{6}{2}}}$
$= \frac{3^{15}}{3^9}$
$= 3^6$
$= 729$

33. $\left(\sqrt[3]{216}\right)^2 = (216^{\frac{1}{3}})^2$
$= 216^{\frac{2}{3}}$
$= (6^3)^{\frac{2}{3}}$
$= 6^2$
$= 36$

34. $81^{\frac{1}{2}} - 81^{-\frac{1}{2}} = (9^2)^{\frac{1}{2}} - \frac{1}{(9^2)^{\frac{1}{2}}}$
$= 9 - \frac{1}{9}$
$= 8\frac{8}{9}$

35. $\frac{1}{\sqrt[7]{(-128)^4}} = \frac{1}{(-128)^{\frac{4}{7}}}$
$= \frac{1}{[(-2)^7]^{\frac{4}{7}}}$
$= \frac{1}{(-2)^4}$
$= \frac{1}{16}$

36. $(3n^2)^3 = 3^3(n^2)^3$
$= 27n^6$

37. $(y^2)^{-4} \cdot y^8 = \frac{1}{(y^2)^4} \cdot y^8$
$= \frac{1}{y^8} \cdot y^8$
$= 1$

38. $(4y^4)^{\frac{3}{2}} = 4^{\frac{3}{2}}(y^4)^{\frac{3}{2}}$
$= (2^2)^{\frac{3}{2}}(y^4)^{\frac{3}{2}}$
$= 2^{\frac{6}{2}}y^{\frac{12}{2}}$
$= 8y^6$

39. $(27p^3q^6r^{-1})^{\frac{1}{3}} = 27^{\frac{1}{3}}(p^3)^{\frac{1}{3}}(q^6)^{\frac{1}{3}}\left(\frac{1}{r}\right)^{\frac{1}{3}}$
$= (3^3)^{\frac{1}{3}}(p^3)^{\frac{1}{3}}(q^6)^{\frac{1}{3}}\left(\frac{1}{r}\right)^{\frac{1}{3}}$
$= 3^{\frac{3}{3}}p^{\frac{3}{3}}q^{\frac{6}{3}}\ \frac{1}{r^{\frac{1}{3}}}$
$= 3pq^2r^{-\frac{1}{3}}$

40. $[(2x^4]^{-2} = \frac{1}{[(2x)^4]^2}$
$= \frac{1}{2^8x^8}$
$= 2^{-8}x^{-8}$ or $\frac{1}{256x^8}$

41. $(36x^6)^{\frac{1}{2}} = 36^{\frac{1}{2}}(x^6)^{\frac{1}{2}}$
$= 6|x|^3$

42. $\left(\frac{b^{2n}}{b^{-2n}}\right)^{\frac{1}{2}} = (b^{2n} \cdot b^{2n})^{\frac{1}{2}}$
$= (b^{4n})^{\frac{1}{2}}$
$= b^{2n}$

43. $\frac{2n}{4n^{\frac{1}{2}}} = \frac{2n \cdot n^{-\frac{1}{2}}}{4}$
$= \frac{2n^{\frac{1}{2}}}{4}$
$= \frac{\sqrt{n}}{2}$

44. $\left(3m^{\frac{1}{2}} \cdot 27n^{\frac{1}{4}}\right)^4 = 3^4\left(m^{\frac{1}{2}}\right)^4 27^4\left(n^{\frac{1}{4}}\right)^4$
$= 3^4m^2(3^3)^4n$
$= 3^{16}m^2n$

45. $\left(\frac{f^{-16}}{256g^4h^{-4}}\right)^{-\frac{1}{4}} = (f^{-16} \cdot 256^{-1} \cdot g^{-4} \cdot h^4)^{-\frac{1}{4}}$
$= (f^{-16})^{-\frac{1}{4}}(256^{-1})^{-\frac{1}{4}}(g^{-4})^{-\frac{1}{4}}(h^4)^{-\frac{1}{4}}$
$= f^4\, 256^{\frac{1}{4}} \cdot |g| \cdot |h|^{-1}$
$= 4\,f^4|g|\,|h|^{-1}$ or $\frac{4\,f^4|g|}{|h|}$

46. $\sqrt[6]{x^2\left(x^{\frac{3}{4}} + x^{-\frac{3}{4}}\right)} = \left[x^2\left(x^{\frac{3}{4}} + x^{-\frac{3}{4}}\right)\right]^{\frac{1}{6}}$
$= \left[x^2 \cdot x^{-\frac{3}{4}}\left(x^{\frac{6}{4}} + 1\right)\right]^{\frac{1}{6}}$
$= \left[x^{\frac{5}{4}}\left(x^{\frac{3}{2}} + 1\right)\right]^{\frac{1}{6}}$

47. $\left(2x^{\frac{1}{4}}y^{\frac{1}{3}}\right)\left(3x^{\frac{1}{4}}y^{\frac{2}{3}}\right) = 6x^{\frac{2}{4}}y^{\frac{3}{3}}$
$= 6x^{\frac{1}{2}}y$

48. $\sqrt[m]{\sqrt[n]{a}} = \left(a^{\frac{1}{n}}\right)^{\frac{1}{m}}$
$= a^{\frac{1}{n}\cdot\frac{1}{m}}$
$= a^{\frac{1}{nm}}$
$= a^{\frac{1}{mn}}$
$= \sqrt[mn]{a}$

49. $\sqrt{m^6n} = (m^6)^{\frac{1}{2}}\left(n^{\frac{1}{2}}\right)$
$= |m|^3n^{\frac{1}{2}}$

50. $\sqrt{xy^3} = (x)^{\frac{1}{2}}(y^3)^{\frac{1}{2}}$
$= |x|^{\frac{1}{2}}|y|^{\frac{3}{2}}$

51. $\sqrt[3]{8x^3y^6} = 8^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^6)^{\frac{1}{3}}$
$= 2xy^2$

52. $17\sqrt[7]{x^{14}y^7z^{12}} = 17(x^{14})^{\frac{1}{7}}(y^7)^{\frac{1}{7}}(z^{12})^{\frac{1}{7}}$
$= 17x^2yz^{\frac{12}{7}}$

53. $\sqrt[5]{a^{10}b^2} \cdot \sqrt[4]{c^2} = (a^{10})^{\frac{1}{5}}(b^2)^{\frac{1}{5}}(c^2)^{\frac{1}{4}}$
$= a^2b^{\frac{2}{5}}|c|^{\frac{1}{2}}$

54. $60\sqrt[8]{r^{80}s^{56}t^{27}} = 60(r^{80})^{\frac{1}{8}}(s^{56})^{\frac{1}{8}}(t^{27})^{\frac{1}{8}}$
$= 60r^{10}|s|t^{\frac{27}{8}}$

55. $16^{\frac{1}{5}} = \sqrt[5]{16}$

56. $(7a)^{\frac{5}{8}}b^{\frac{3}{8}} = \sqrt[8]{7^5a^5b^3}$

57. $p^{\frac{2}{3}}q^{\frac{1}{2}}r^{\frac{1}{3}} = p^{\frac{4}{6}}q^{\frac{3}{6}}r^{\frac{2}{6}}$
$= \sqrt[6]{p^4q^3r^2}$

58. $\frac{2^{\frac{2}{3}}}{2^{\frac{1}{3}}} = 2^{\frac{1}{3}}$
$= \sqrt[3]{2}$

59. $13a^{\frac{1}{7}}b^{\frac{1}{3}} = 13a^{\frac{3}{21}}b^{\frac{7}{21}}$
$= 13\sqrt[21]{a^3b^7}$

60. $(n^3m^9)^{\frac{1}{2}} = (n^3)^{\frac{1}{2}}(m^9)^{\frac{1}{2}}$
$= n^{\frac{3}{2}}m^{\frac{9}{2}}$
$= |n|\,m^4\sqrt{mn}$

61. $x = \sqrt[3]{(-245)^{-\frac{1}{5}}}$
$\approx \sqrt[3]{-0.33}$
≈ -0.69

62. $d^3e^2f^2 = (d^3)^{\frac{1}{2}}(e^2)^{\frac{1}{2}}(f^2)^{\frac{1}{2}}$
$= d|e|\,|f|\sqrt{d}$

63. $\sqrt[3]{a^5b^7c} = (a^5)^{\frac{1}{3}}(b^7)^{\frac{1}{3}}(c)^{\frac{1}{3}}$
$= ab^2\sqrt[3]{a^2bc}$

64. $\sqrt{20x^3y^6} = (20)^{\frac{1}{2}}(x^3)^{\frac{1}{2}}(y^6)^{\frac{1}{2}}$
$= 2x|y|^3\sqrt{5x}$

65. $14.2 = x^{-\frac{3}{2}}$
$(14.2)^{-\frac{2}{3}} = (x^{-\frac{3}{2}})^{-\frac{2}{3}}$
$0.17 \approx x$

66. $724 = 15a^{\frac{5}{2}} + 12$
$712 = 15a^{\frac{5}{2}}$
$\frac{712}{15} = a^{\frac{5}{2}}$
$\left(\frac{712}{15}\right)^{\frac{2}{5}} = (a^{\frac{5}{2}})^{\frac{2}{5}}$
$4.68 \approx a$

67. $\frac{1}{8}\sqrt{x^5} = 3.5$
$x^{\frac{5}{2}} = 28$
$(x^{\frac{5}{2}})^{\frac{2}{5}} = (28)^{\frac{2}{5}}$
$x \approx 3.79$

68. $d = 6.794 \times 10^3$ km so $r = 3.397 \times 10^3$ km
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(3.397 \times 10^3 \text{ km})^3$
$\approx 1.64 \times 10^{11} \text{ km}^3$

69. $y = 3^x$; $x = -8, -6, -5, \frac{10}{33}, \frac{1}{2}, \frac{2}{3}, \frac{10}{9}, \frac{5}{3}, \frac{7}{2}$

x	3^x	y
-8	3^{-8}	$\frac{9}{6561}$
-6	3^{-6}	$\frac{1}{729}$
-5	3^{-5}	$\frac{1}{243}$
$\frac{10}{33}$	$3^{\frac{10}{33}}$	1.395
$\frac{1}{2}$	$3^{\frac{1}{2}}$	1.732
$\frac{2}{3}$	$3^{\frac{2}{3}}$	2.080
$\frac{10}{9}$	$3^{\frac{10}{9}}$	3.389
$\frac{5}{3}$	$3^{\frac{5}{3}}$	14.620
$\frac{7}{2}$	$3^{\frac{7}{2}}$	46.765

69a. If $x < 0$ then $y > 0$. If $x = 0$ then $y = 1$. Since $x < 0$, $y > 0$ and $y < 1$. So, $0 < y < 1$.

69b. If $x > 0$ then $y > 1$. If $x < 1$ then $y < 3$. So, $1 < y < 3$.

69c. If $x > 1$ then $y > 3$. So, $y > 3$.

69d. If the exponent is less than 0, the power is greater than 0 and less than 1. If the exponent is greater than 0 and less than 1, the power is greater than 1 and less than the base. If the exponent is greater than 1, the power is greater than the base.

Any number to the zero power is 1. Thus, if the exponent is less than zero, the power is less than 1. A power of a positive number is never negative, so the power is greater than 0.

Any number to the zero power is 1 and to the first power is itself. Thus, if the exponent is greater than zero and less than 1, the power is between 1 and the base.

Any number to the first power is itself. Thus, if the exponent is greater than 1, the power is greater than the base.

70. $r = (1.2 \times 10^{-15})A^{\frac{1}{3}}$

If $r = 2.75 \times 10^{-15}$ then

$$2.75 \times 10^{-5} = (1.2 \times 10^{-15})A^{\frac{1}{3}}$$
$$\frac{2.75 \times 10^{-15}}{1.2 \times 10^{-15}} = A^{\frac{1}{3}}$$
$$2.29 \approx A^{\frac{1}{3}}$$
$$12.04 \approx A$$

Since $12.04 \approx 12$, which is the mass number of carbon, the atom is carbon.

71.
$$32^{(x^2+4x)} = 16^{(x^2+4x+3)}$$
$$(2^5)^{(x^2+4x)} = (2^4)^{(x^2+4x+3)}$$
$$2^{(5x^2+20x)} = 2^{(4x^2+16x+12)}$$
$$5x^2 + 20x = 4x^2 + 16x + 12$$
$$x^2 + 4x - 12 = 0$$
$$(x + 6)(x - 2) = 0$$
$$x + 6 = 0 \qquad x - 2 = 0$$
$$x = -6 \qquad x = 2$$

72a.

Wind Speed	Wind Chill
5	0.8
10	−12.7
15	−22.6
20	−29.9
25	−35.3
30	−39.2

72b. A 5-mile per hour increase in the wind speed when the wind is light has more of an effect on perceived temperature than a 5-mile per hour increase in the wind speed when the wind is heavy.

73a. $r^3 = \frac{GM_e t^2}{4\pi^2}$ $\qquad G = 6.67 \times 10^{-11}$

$M_t = 5.98 \times 10^{24}$

$t = 1$ day $= 86{,}4000$ seconds

$$r^3 = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(86{,}400)^2}{4\pi^2}$$
$$r \approx 42{,}250{,}474.31 \text{ m}$$

73b. $42{,}250{,}474.31 \text{ m} = 42{,}250.47431 \text{ km}$

$42.250.47431 - 6380 = 35870.47431$

$\approx 35{,}870 \text{ km}$

74a. $a^m a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}} \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m+n \text{ factors}} = a^{m+n}$

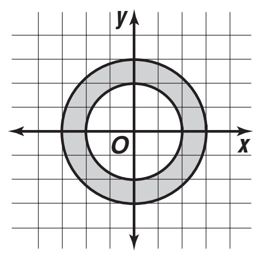
74b. $(a^m)^n = \overbrace{\overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}} \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}} \cdot \ldots \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}}}^{n \text{ factors}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \cdot n \text{ factors}} = a^{m+n}$

74c. $(ab)^m = \overbrace{ab \cdot ab \cdot \ldots \cdot ab}^{m \text{ factors}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}} \cdot \overbrace{b \cdot b \cdot \ldots \cdot b}^{m \text{ factors}} = a^m b^m$

74d. $\left(\frac{a}{b}\right)^m = \overbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \ldots \cdot \frac{a}{b}}^{m \text{ factors}} = \frac{a^m}{b^m}$

74e. $\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}}}{\overbrace{a \cdot a \cdot \ldots \cdot a}^{n \text{ factors}}} \; a^{m-n}$

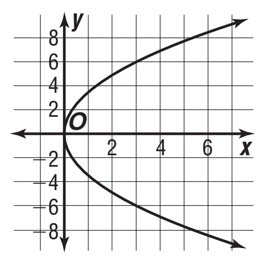
75.



76. $y^2 = 12x$

$(y - 0)^2 = 4(3)(x - 0)$

Vertex is at (0, 0) and $p = 3$. The parabola opens to the right so the focus is at (0 + 3, 0) or (3, 0). Since the directrix is 3 units to the left of the vertex, the equation of the directrix is $x = -3$.



77. $(2\sqrt{3} + 2i)^{\frac{1}{5}}$

Convert to polar form $r(\cos\theta + \boldsymbol{i}\sin\theta)$.

$r = \sqrt{\left(2\sqrt{3}\right)^2 + 2^2}$ $\qquad \theta = \text{Arctan}\,\frac{2}{2\sqrt{3}}$

$= \sqrt{16}$ $\qquad = \text{Arctan}\,\frac{\sqrt{3}}{3}$

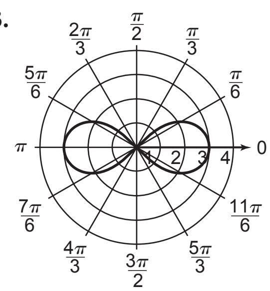
$= 4$ $\qquad = \frac{\pi}{6}$

So, $(2\sqrt{3} + 2\boldsymbol{i}) = 4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)$.

Use De Moivre's Theorem.

$$\left[4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)\right]^{\frac{1}{5}} = 4^{\frac{1}{5}}\left(\cos\left(\frac{1}{5}\cdot\frac{\pi}{6}\right) + \sin\left(\frac{1}{5}\cdot\frac{\pi}{6}\right)\right)$$
$$= 4^{\frac{1}{5}}\cos\frac{\pi}{30} + 4^{\frac{1}{5}}\boldsymbol{i}\sin\frac{\pi}{30}$$
$$= 1.31 + 0.14\boldsymbol{i}$$

78.



Lemniscate

79. Use the equation $y = t|\vec{v}|\sin\theta - \frac{1}{2}gt^2 + h$.

$|\vec{v}| = 105 \qquad g = 32 \qquad h = 3 \qquad \theta = 42$

$y = t(105)(\sin 42°) - \frac{1}{2}(32)t^2 + 3$

$\quad = 16t^2 - (105 \sin 42°)t - 3$

Find t when $y = 0$ (i.e., the ball is on the ground).

$t = \frac{105 \sin 42° \pm \sqrt{(105 \sin 42°) - 4(16)(-3)}}{2(16)}$

$t = -0.04, 4.43$

So, the ball hits the ground after about 4.43 s.

80. $\overrightarrow{TC} = \langle (2-3), (6-(-4)), (-5-6) \rangle$

$\quad = \langle -1, 10, -11 \rangle$

$|\overrightarrow{TC}| = \sqrt{(2-3)^2 + (6-(-4))^2 + (-5-6)^2}$

$\quad = \sqrt{222}$

81. Sample answer:

$\tan S \cos S = \frac{1}{2}$

$\frac{\sin S}{\cos S} \cdot \frac{\cos S}{1} = \frac{1}{2}$

$\sin S = \frac{1}{2}$

82. $\cot\theta = 0$

$\theta = \frac{\pi}{2}$

Period of $\cot\theta$ is π so

$\cot\theta = 0$

$\theta = \frac{\pi}{2} + \pi n$, where n is an integer.

83. $r \cdot 6h = 150\pi$ m

$r = \frac{150\pi \text{ m}}{6 \text{ h}}$

$r = 25\pi$ m/h

84. 90°, 270°

85. $x^3 - 25x = 0$

Degree of 3 so there are 3 complex roots.

$x^3 - 25x = 0$

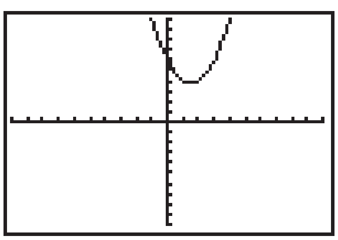
$x(x-5)(x+5) = 0$

$x = 0 \qquad x - 5 = 0 \qquad x + 5 = 0$

$\qquad\qquad x = 5 \qquad x = 5$

3; −5, 0, 5

86.



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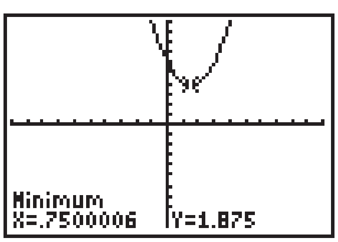


abs. min; (0.75, 1.88)

87. The time it takes to paint a building is inversely proportional to the number of painters.

$48 = \frac{k}{8}$

$k = 384$

So $t = \frac{384}{16}$

$t = 24$

The correct choice is E.

11-2 Exponential Functions

Page 705 Graphing Calculator Exploration

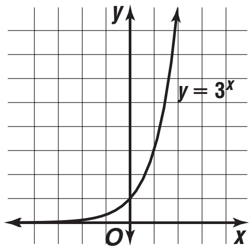
1. positive reals
2. (0, 1)
3. For $a = 0.5$ and 0.75, $y \to \infty$ as $x \to -\infty$, $y \to 0$ as $x \to \infty$. For $a = 2$ and 5, $y \to 0$ as $x \to -\infty$, $y \to \infty$ as $x \to \infty$.
4. horizontal asymptote at $y = 0$, no vertical asymptotes
5. Yes; the range of an exponential function is all positive reals because the value of any positive real number raised to any power is positive.
6. Any nonzero number raised to the zero power is 1.
7. The graph of $y = b^x$ is decreasing when $0 < b < 1$ because multiplying by number between 0 and 1 results in a product less than the original number. The graph of $y = b^x$ is increasing when $b > 1$ because multiplying by a number greater than 1 results in a product greater than the original number.
8. There is a horizontal asymptote at $y = 0$ because a power of a positive real number is never 0 or less. As a number between 0 and 1 is raised to greater and greater powers, its value approaches 0. As a number greater than 1 is raised to powers approaching negative infinity, its value approaches 0.

Page 708 Check for Understanding

1. Power; in a power function the variable is the base, in an exponential function the variable is the exponent.
2. Both graphs are one-to-one, have the domain of all reals, a range of positive reals, a horizontal asymptote of $y = 0$, a y-intercept of (0, 1), and no vertical asymptote. The graph of $y = b^x$ is decreasing when $0 < b < 1$ and increasing when $b > 1$.
3. If the base is greater than 1, the equation represents exponential growth. If base is between 0 and 1, the equation represents exponential decay.
4. The graphs of $y = 4^x$ and $y = 4^x - 3$ are the same except the graph of $y = 4^x - 3$ is shifted down three units from the graph of $y = 4^x$.

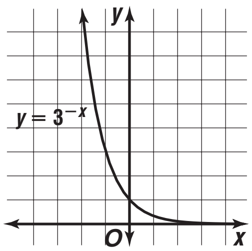
5.

x	y
-1	$\frac{1}{9}$
0	1
1	3
2	9



6.

x	y
-2	9
-1	3
0	1
1	$\frac{1}{9}$



7.

x	y
-2	$-3\frac{3}{4}$
-1	$-3\frac{1}{2}$
0	-4
1	-2
2	0

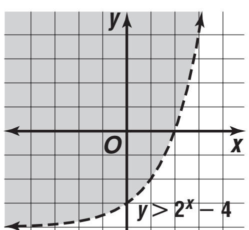
Use (0, 0) as a test point.

$0 \overset{?}{>} 2^0 - 4$

$0 \overset{?}{>} 1 - 4$

$0 > -3$ ✓

The statement is true so shade the region containing (0, 0).



8. Use $N = N_0\,(1 + r)^t$ where $N_0 = 3750$, $r = -0.25$, and $t = 2$.

$N = 3750\,(1 - 0.25)^2$

$= 3750\,(0.75)^2$

≈ 2109.38

9a. $9{,}145{,}219 - 8{,}863{,}052 = 282{,}167$

$\frac{282{,}167}{7} = 40{,}309.57$

$\frac{40{,}309.57}{8{,}863{,}052} \approx 0.0045$

$\approx 0.45\%$

9b. Use $N = N_0(1 + r)^t$.

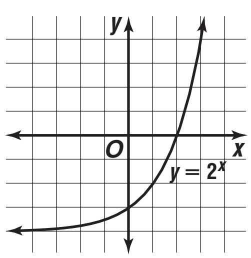
$N \approx 8{,}863{,}052(1 + 0.0045)^{20}$

$N \approx 9{,}695{,}766$

Pages 708–711 Exercises

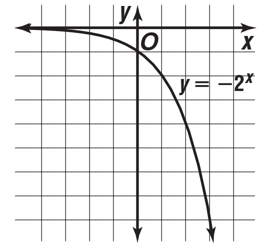
10.

x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4



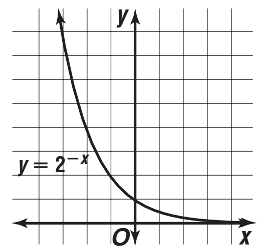
11.

x	y
-1	$-\frac{1}{2}$
0	-1
1	-2
2	-4



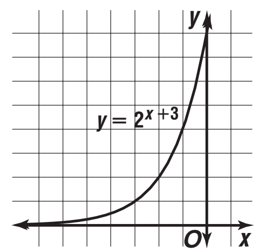
12.

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$



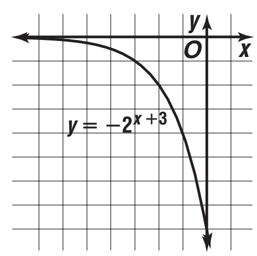
13.

x	y
-3	1
-2	2
-1	4
0	8



14.

x	y
-3	-1
-2	-2
-1	-4
0	-8



15.

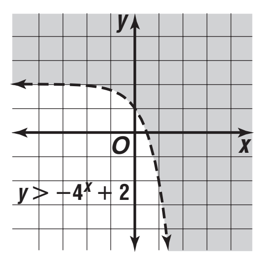
x	y
-1	$1\frac{3}{4}$
0	1
1	-2
2	-14

(0, 0)

$0 \overset{?}{>} -4^2 + 2$

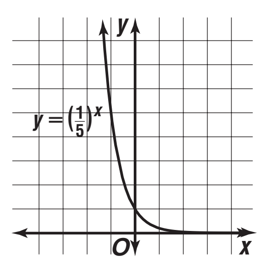
$0 \overset{?}{>} -1 + 2$

$0 \not> 1$; no



16.

x	y
-1	5
0	1
1	$\frac{1}{5}$



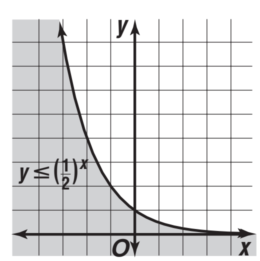
17.

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$

(0, 0)

$0 \overset{?}{\leq} \left(\frac{1}{2}\right)^0$

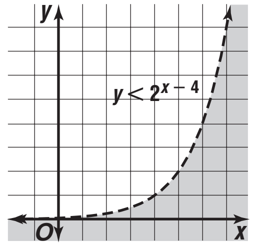
$0 \overset{?}{\leq} 1$ ✓



18.

x	y
3	$\frac{1}{2}$
4	1
5	2
6	4

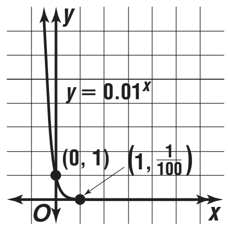
(0, 0)
$0 \overset{?}{<} 2^{0-4}$
$0 \overset{?}{<} 2^{-4}$
$0 < \frac{1}{16}$ ✓



19.

x	y
−1	100
0	1
1	$\frac{1}{100}$

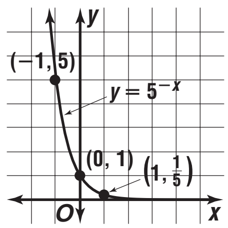
B



20.

x	y
−1	5
0	1
1	$\frac{1}{5}$

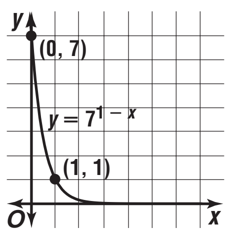
C



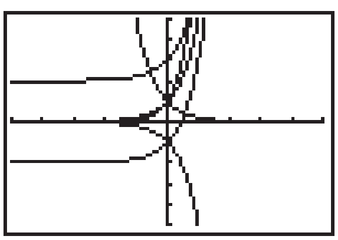
21.

x	y
−1	49
0	7
1	1

A



22.



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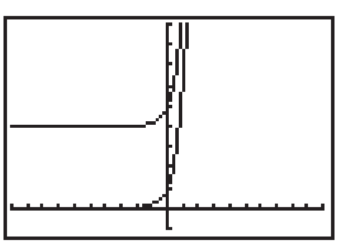
22a. The graph of $y = -5^x$ is a reflection of $y = 5^x$ across the x-axis. The graph of $y = 5^x$ is a reflection of $y = 5^x$ across the y-axis.

22b. The graph of $y = 5^x + 2$ is shifted up two units, while the graph of $y = 5^x - 2$ is shifted down two units.

22c. The graph of $y = 10^x$ increases more quickly than the graph of $y = 5^x$. The graphs are not the same because $5^{2x} \neq 10^x$.

23a. The graph of $y = 6^x + 4$ is shifted up four units from the graph of $y = 6^x$.



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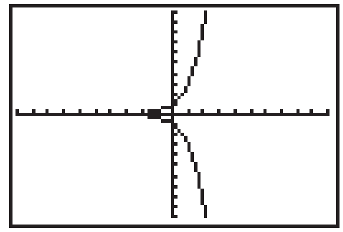
23b. The graph of $y = -3^x$ is a reflection of the graph of $y = 3^x$ across the x-axis.



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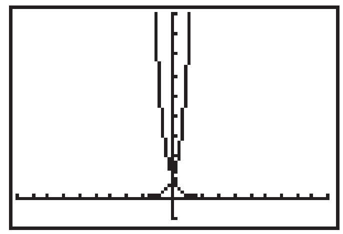
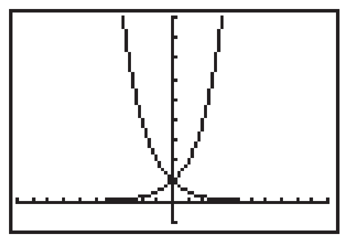
23c. The graph of $y = 7^{-x}$ is a reflection of the graph of $y = 7^x$ across the y-axis.



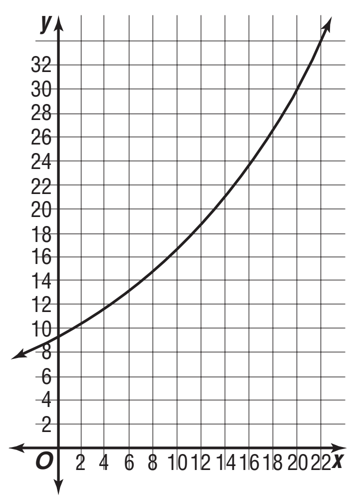
[−10, 10] scl:1 by [−1, 9] scl:1

23d. The graph of $y = \left(\frac{1}{2}\right)^x$ is a reflection of the graph of $y = 2^x$ across the y-axis.



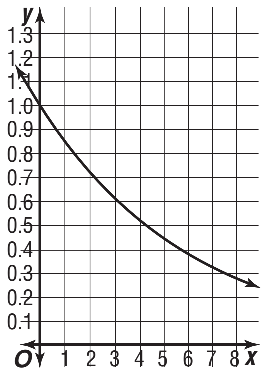
[−10, 10] scl:1 by [−1, 9] scl:1

24a.



24b. $y = 9.25(1.06)^{50}$
$y \approx 170.386427$ thousand
$y \approx \$170{,}400$

25a. $y = (0.85)^x$

25b. 

25c. $y = (0.85)^{12}$
≈ 0.14 or 14%

25d. No; the graph has an asymptote at $y = 0$, so the percent of impurities, y, will never reach 0.

26a. $N = 876{,}156(1 + 0.0074)^{15}$
$\approx 978{,}612.2261$ or $978{,}612$

26b. $N = 2{,}465{,}326(1 - 0.0053)^{15}$
$\approx 2{,}668{,}760.458$ or $2{,}668{,}760$

26c. $152{,}307 = 139{,}510(1 + r)^{10}$
$\frac{152{,}307}{139{,}510} = (1 + r)^{10}$
$\left(\frac{152{,}307}{139{,}510}\right)^{\frac{1}{10}} = 1 + r$
$N = 139{,}510(1 + r)^{25}$
$N = 173{,}736.7334$ or $173{,}737$

26d. $191{,}701 = 168{,}767(1 + r)^{10}$
$\frac{191{,}701}{168{,}767} = (1 + r)^{10}$
$\left(\frac{191{,}701}{168{,}767}\right)^{\frac{1}{10}} = 1 + r$
$N = 168{,}767(1 + r)^{25}$
$N = 232{,}075.6889$ or $232{,}076$

27a. $O = 100(3^{\frac{3 \cdot 5}{5}})$
$= 100(3^3)$
$= 2700$ units

27b. $s = \frac{4.2 \text{ mi}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}}$
$= 6.16$ ft/s
$O = 100(3^{\frac{3 \cdot 6.16}{5}})$
≈ 5800.16
≈ 5800 units

28a. P_n 121,000, $n = 30 \cdot 12$ or 360, $i = 0.075 \div 12$ or 0.00625
$121{,}000 = P\left[\frac{1 - (1 + 0.00625)^{-360}}{0.00625}\right]$
$P \approx$
846.04955; \$846.05

28b. $P_n = 121{,}000$, $n = 20 \cdot 12$ or 240, $i = 0.0725 \div 12$ or 0.00604
$$121{,}000 = P\left[\frac{1 - \left(1 + \frac{0.0725}{12}\right)^{-240}}{\left(\frac{0.0725}{12}\right)}\right]$$
$P \approx 956.35494$; \$956.35

28c. 30 year: $I = 360(846.05) - 121{,}000$
$= \$183.578$
20 year: $I = 240(956.35) - 121{,}000$
$= \$108{,}524$

28d. Sample answer: A borrower might choose the 30-year mortgage in order to have a lower monthly payment. A borrower might choose the 20-year mortgage in order to have a lower interest expense.

29a. $P = 4000$, $n = 43$, $i = 0.0475$
$F_n = 4000\left[\frac{(1 + 0.0475)^{43} - 1}{0.0475}\right]$
$\approx 535{,}215.918$; \$535,215.92

29b. $P = 4000$, $n = 43$, $i = 0.0525$
$F_n = 4000\left[\frac{(1 + 0.0525)^{43} - 1}{0.0525}\right]$
$\approx 611{,}592.1194$ or \$611,592.12
$\$611{,}592.12 - 535{,}215.92 = \$76{,}376.20$

30. The function $y = a^x$ is undefined when $a < 0$ and the exponent x is a fraction with an even denominator.

31a. Compounded once:
$I = 1000[(1 + 0.05)^1 - 1]$
$= 50$; \$50
Compounded twice:
$I = 1000[\left(1 + \frac{0.05}{2}\right)^2 - 1]$
≈ 50.625; \$50.63
Compounded four times:
$I = 1000[\left(1 + \frac{0.05}{4}\right)^4 - 1]$
≈ 50.9453; \$50.94
Compounded twelve times:
$I = 1000\left[\left(1 + \frac{0.05}{12}\right)^{12} - 1\right]$
≈ 51.1619; \$51.16
Compounded 365 times:
$I = 1000\left[\left(1 + \frac{0.05}{365}\right)^{365} - 1\right]$
≈ 51.2675; \$51.26

31b. Let x represent the investment.
Statement savings: $I = x[(1 + 0.051)^1 - 1]$
$= 0.051x$
The return is 5.1%
Money Market Savings: $I = x\left[\left(1 + \frac{0.0505}{12}\right)^{12} - 1\right]$
$= 0.517x$
The return is 5.17%
Super Saver: $I = x\left[\left(1 + \frac{0.05}{365}\right)^{365} - 1\right]$
$= 0.513x$
The return is 5.13%
Money Market Savings

31c. $(1 + 0.05) = \left(1 + \frac{x}{365}\right)^{365}$
$(1.05)^{\frac{1}{365}} = 1 + \frac{x}{365}$
$365[(1.05)^{\frac{1}{365}} - 1] = x$
$0.04879 = x$; 4.88%

32. $4x^2(4x)^{-2} = 4x^2(4)^{-2}(x)^{-2}$
$= \frac{4x^2}{16x^2}$
$= \frac{1}{4}$

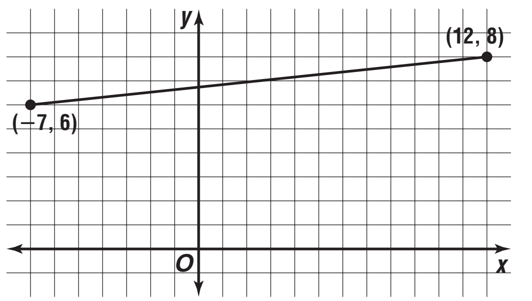
33. $y = 15$
Use $y = r \sin \theta$.
$y = 15$
So, $15 = r \sin \theta$.

34. $\langle -3, 9\rangle - \langle 2, 1\rangle = (-3)(2) + (9)(1)$
$= 3$
3; no because the inner product does not equal 0.

35. $\frac{1}{3}\left(\cos\frac{7\pi}{8} + \boldsymbol{i}\sin\frac{7\pi}{8}\right) \cdot 3\sqrt{3}\left[\cos\left(-\frac{\pi}{4}\right) + \boldsymbol{i}\sin\left(-\frac{\pi}{4}\right)\right]$
$= \left(\frac{1}{3} \cdot 3\sqrt{3}\right)\left[\cos\left(\frac{7\pi}{8} - \frac{\pi}{4}\right) + \boldsymbol{i}\sin\left(\frac{7\pi}{8} - \frac{\pi}{4}\right)\right]$
$= \sqrt{3}\left(\cos\frac{5\pi}{8} + \boldsymbol{i}\sin\frac{5\pi}{8}\right)$
$= -0.66 + 1.60\boldsymbol{i}$

36. $s = 72t - 16t^2 + 4$
$s - 4 = -16t^2 + 72t$
$s - 4 + (-16)(5.0625) = -16(t^2 - 4.5 + 5.0625)$
$(s - 85) = -16(t - 2.25)^2$
Vertex: (2.25, 85)
Maximum height: 85 feet.

37a.

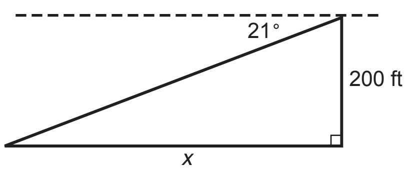


37b. $\left(\frac{(12 + (-7))}{2}, \frac{(8 + 6)}{2}\right) = (2.5, 7)$

38.
$\sin^4 A + \cos^2 A = \cos^4 A + \sin^2 A$
$(\sin^2 A)^2 + \cos^2 A = \cos^4 A + \sin^2 A$
$(1 - \cos^2 A)^2 + \cos^2 A = \cos^4 A + \sin^2 A$
$(1 - 2\cos^2 A + \cos^4 A) + \cos^2 A = \cos^4 A + \sin^2 A$
$\cos^4 A + 1 - \cos^2 A = \cos^4 A + \sin^2 A$
$\cos^4 A + \sin^2 A = \cos^4 A + \sin^2 A$

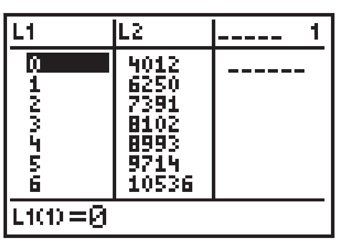
39. $\frac{2900 \text{ rev}}{1} \cdot \frac{2\pi}{1 \text{ rev}} = 4800\pi$
$V = 9.2\,\frac{4800\pi}{1}$
$\approx 138{,}732.73$ or about 139,000 cm/s

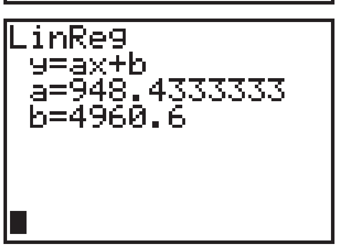
40.



$\tan 69° = \frac{x}{200}$
$200 \tan 69° = x$
$521.02 = x$
about 521 feet

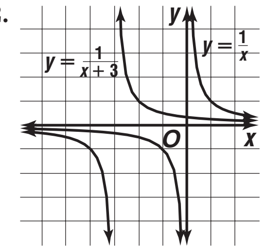
41.





Sample answer: $y = 948.4x + 4960.6$

42.



The parent graph is translated 3 units left. The vertical asymptote is now $x = -3$. The horizontal asymptote, $y = 0$, is unchanged.

43. $C_{AC} = 32\pi \approx 100.53$ $\quad C_{AB} = 16\pi \approx 50.27$
$100.53 - 50.27 = 50.26$ or about 50.
The correct choice is E.

11-3 The Number *e*

Page 714 Check for Understanding

1. C
2. If k is positive, the equation models growth. If k is negative, the equation models decay.
3. Amount in an account with a beginning balance of \$3000 and interest compounded continuously at an annual rate of 5.5%.
4. reals, positive reals
5. Sample answer: Continuously compounded interest is a continuous function, but interest compounded monthly is a discrete function.

6a. growth
6b. 33,430
6c. $y = 33{,}430e^{0.0397(60)}$
$\approx 361{,}931.0414$ or 361,931

7. $A = 12{,}000e^{0.064(12)}$
$\approx 25{,}865.412$ or \$25,865.41

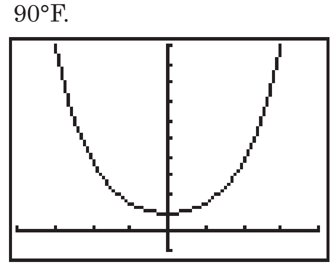
Pages 714–717 Exercises

8. $p = (100 - 18)e^{-06(2)} + 18$
$= 42.6$ or 43%

9a. $y = 84e^{-0.23(15)} + 76$
$= 78.66$ or 78.7°F

9b. too cold; After 5 minutes, his coffee will be about 90°F.

10a.



[−4, 4] scl:1 by [−1, 10] scl:1

10b. symmetric about y-axis

11a. Annually: $I = 100[(1 + 0.08)^1 - 1]$
$= 80$ $80.00; 8%

Semi-annually: $I = 1000\left[\left(1 + \frac{0.08}{2}\right)^2 - 1\right]$
$= 81.6$ $81.60; 8.16%

Quarterly: $I = 1000\left[\left(1 + \frac{0.08}{4}\right)^4 - 1\right]$
≈ 82.4316 $82.43; 8.243%

Monthly: $I = 1000\left[\left(1 + \frac{0.08}{12}\right)^{12} - 1\right]$
≈ 82.9995 $83.00; 8.3%

Daily: $I = 1000\left[\left(1 + \frac{0.08}{365}\right)^{365} - 1\right]$
≈ 83.2776 $83.28; 8.328%

Continuously: $I = 1000(e^{0.08(1)} - 1)$
≈ 83.2871 $83.98; 8.329%

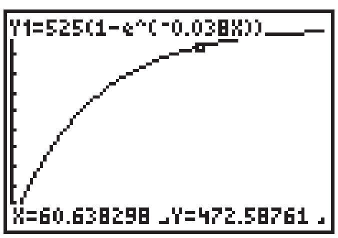
Interest Compounded	Interest	Effective Annual Yield
Annually	$80.00	8%
Semi-annually	$81.60	8.16%
Quarterly	$82.43	8.243%
Monthly	$83.00	8.3%
Daily	$83.28	8.328%
Continuously	$83.29	8.329%

11b. continuously **11c.** $E = \left(1 + \frac{r}{n}\right)^n - 1$

11d. $E = e^r - 1$

12a. $y = 525(1 - e^{-0.038(24)})$
≈ 314.097 314 people

12b.

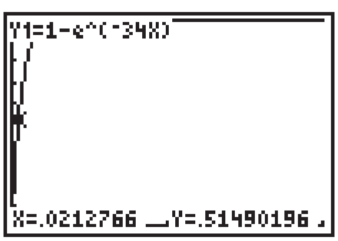


after about 61h

[0, 100] scl:10 by [0, 550] scl:50

13a. $P = 1 - e^{-6(0.5)}$
≈ 0.95021 95%

13b.



$x = 0.02$; about 0.02 h

$\frac{0.02\text{h}}{1} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 1.2$

[0, 1] scl:0.1 by [0, 1] scl:0.1 about 1.2 min

14a. For $x = 10$: $\left(\frac{2(10) + 1}{2(10) - 1}\right)^{10} = \left(\frac{21}{19}\right)^{10}$
≈ 2.720551414

For $x = 100$: $\left(\frac{2(100) + 1}{2(100) - 1}\right)^{100} = \left(\frac{201}{199}\right)^{100}$
≈ 2.718304481

For $x = 1000$: $\left(\frac{2(1000) + 1}{2(1000) - 1}\right)^{1000} = \left(\frac{2001}{1999}\right)^{1000}$
≈ 2.718282055

2.720551414; 2.718304481; 2.718282055

14b. 2 decimal places; 4 decimal places; 6 decimal places

14c. always greater

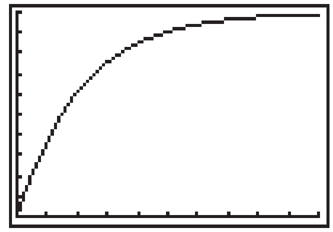
15a. 5 days: $P = 1 - e^{-0.047(5)}$
≈ 0.20943 20.9%

20 days: $P = 1 - e^{-0.047(26)}$
≈ 0.60937 60.9%

90 days: $P = 1 - e^{-0.047(90)}$
≈ 0.98545 98.5%

20.9%; 60.9% 98.5%

15b.



about 29 days

15c. Sample answer: The probability that a person who is going to respond has responded approaches 100% as t approaches infinity. New ads may be introduced after a high percentage of those who will respond have responded. The graph appears to level off after about 50 days. So, new ads can be introduced after an ad has run about 50 days.

16a. all reals

16b. $0 < f(x) < 1$

16c. c shifts the graph to the right or left

17. $120{,}000 = P\left[\frac{(1 + 0.035)^8 - 1}{0.035}\right]$

$120{,}000 \approx P(9.051687)$

$P \approx 13{,}257.19725$

$13,257.20

18. $x^{\frac{8}{5}}y^{\frac{3}{5}}z^{\frac{1}{5}} = x\sqrt[5]{x^3y^3z}$

19. $y = 6x^2$ $\theta = 45°$

$-x \sin 45° + y \cos 45° = 6(x \cos 45° + y \sin 45°)^2$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 6\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 6\left(\frac{1}{2}x^2 + xy + \frac{1}{2}y^2\right)$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 3x^2 + 6xy + 3y^2$

$-\sqrt{2}x + \sqrt{2}y = 6x^2 + 12xy + 6y^2$

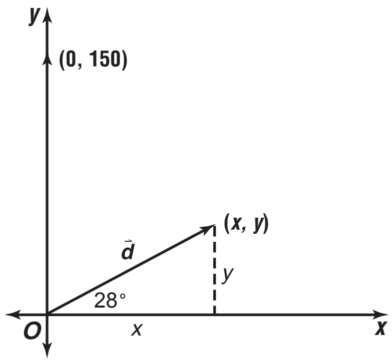
$6x^2 + 12xy + 6y^2 + \sqrt{2}x - \sqrt{2}y = 0$

20. $r = \sqrt{(-5)^2 + (-1)^2}$ $\theta = \text{Arctan}\,\frac{-1}{-5} + \pi$

$= \sqrt{26}$ ≈ 3.34

$\sqrt{26}(\cos 3.34 + \boldsymbol{i} \sin 3.34)$

21.



$\vec{d} = \langle x, y\rangle \qquad \vec{F} = \langle 0, 150\rangle$

$\cos 28° = \frac{x}{10} \qquad \sin 28° = \frac{x}{10}$

$x = 10\cos 28° \qquad x = 10\sin 28°$

$x \approx 8.8295 \qquad x \approx 4.6947$

$\boldsymbol{d} = \langle 8.8295, 4.6947\rangle$

$W = \vec{F} \cdot \vec{d}$

$W = \langle 0, 150\rangle \cdot \langle 8.8295, 4.6947\rangle$

$= 0 + 704.205$

$= 704.2$ ft-lb

22. $y = -1.5(10)^2 + 13.3(14) + 19.4$

$= 2.4$

23. $\sqrt{2x+3} = 4$

$2x + 3 = 16$

$2x = 13$

$x = \frac{13}{2}$

24. $|3x + 2| \le 6$

$3x + 2 \le 6 \qquad 3x + 2 \ge -6$

$3x \le 4 \qquad 3x \ge -8$

$x \le \frac{4}{3} \qquad x \ge -\frac{8}{3}$

$\left\{x \mid -\frac{8}{3} \le x \le \frac{4}{3}\right\}$

25. $3\begin{bmatrix} -3 & -2 & 2 & 3 \\ -2 & 6 & 5 & -1 \end{bmatrix} = \begin{bmatrix} -9 & -6 & 6 & 9 \\ -6 & 18 & 15 & -3 \end{bmatrix}$

$J'(-9, -6)$, $K'(-6, 18)$, $L'(6, 15)$, $M'(9, -3)$; The dilated image has sides that are 3 times the length of the original figure.

26. $\begin{bmatrix} 4x + y \\ x \end{bmatrix} = \begin{bmatrix} 6 \\ 2y - 12 \end{bmatrix}$

$4x + y = 6 \qquad 4(2y - 12) + y = 6 \qquad x = 2(6) - 12$

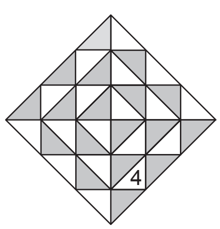
$x = 2y - 12 \qquad 9y = 54 \qquad x = 0$

$y = 6$

$(0, 6)$

27. $\{-4, 2, 5\}$; $\{5, 7\}$; yes

28.



The correct choice is D.

Page 717 Mid-Chapter Quiz

1. $64^{\frac{1}{2}} = 8$

2. $(\sqrt[3]{343})^{-2} = (343^{\frac{1}{3}})^{-2}$

$= ((7^3)^{\frac{1}{3}})^{-2}$

$= 7^{-2}$

$= \frac{1}{49}$

3. $\left(\frac{8x^3y^{-6}}{27w^6z^{-9}}\right)^{\frac{1}{3}} = \left(\frac{8x^3z^9}{27w^6y^6}\right)^{\frac{1}{3}}$

$= \frac{(2^3)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(z^9)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}(w^6)^{\frac{1}{3}}(y^6)^{\frac{1}{3}}}$

$= \frac{2xz^3}{3w^2y^2}$ or $\frac{2}{3}w^{-2}xy^{-2}z^3$

4. $\sqrt{a^6b^3} = (a^4b^3)^{\frac{1}{2}}$

$= (a^6)^{\frac{1}{2}}(b^3)^{\frac{1}{2}}$

$= |a|^3 b^{\frac{3}{2}}$

5. $(125a^2b^3)^{\frac{1}{3}} = \sqrt[3]{125a^2b^3}$

$= \sqrt[3]{5^3a^2b^3}$

$= 5b\sqrt[3]{a^2}$

6. $1.75 \times 10^2 = 0.094\sqrt{A^3}$

$1.75 \times 10^2 = 0.94\,A^{\frac{3}{2}}$

$\frac{1.75 \times 10^2}{0.94} = A^{\frac{3}{2}}$

$\left(\frac{1.75 \times 10^2}{0.94}\right)^{\frac{2}{3}} = (A^{\frac{3}{2}})^{\frac{2}{3}}$

$151.34 \approx A$

1.51×10^2 mm^2

7. $1{,}786{,}691 = 1{,}637{,}859\,(1 + r)^8$

$\frac{1{,}786{,}691}{1{,}637{,}859} = (1 + r)^8$

$\left(\frac{1{,}786{,}691}{1{,}637{,}859}\right)^{\frac{1}{8}} = [(1 + r)^8]^{\frac{1}{8}}$

$\left(\frac{1{,}786{,}691}{1{,}637{,}859}\right)^{\frac{1}{8}} - 1 = r$

$r \approx 0.011$ Store the exact value in your calculator's memory.

$N = 1{,}637{,}859\,(1 + 0.011)^{24}$ Use the stored value for r.

$= 2{,}216{,}156.979$

$2{,}126{,}157$

8. $A = 3500\left(1 + \frac{0.052}{4}\right)^{(4)(3.5)}$

$= 3500(1.013)^{14}$

$= 4193.728$

$\$4193.73$

9a. $y = 6.7e^{\frac{-48.1}{15}}$

≈ 0.271292

$271{,}292$ ft^3

9b. $y = 6.7e^{\frac{-48.1}{50}}$

≈ 2.560257

$2{,}560{,}257$ ft^3

10. 2 years: $n = \frac{200}{1 + 20e^{-0.35(2)}}$

≈ 18.3

15 years: $n = \frac{200}{1 + 20e^{-0.35(15)}}$

≈ 181

60 years: $n = \frac{200}{1 + 20e^{-0.35(60)}}$

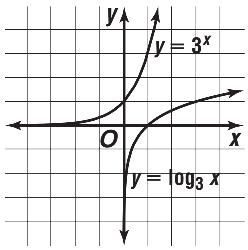
≈ 200

18.3; 181; 200

11-4 Logarithmic Functions

Pages 722–723 Check for Understanding

1.

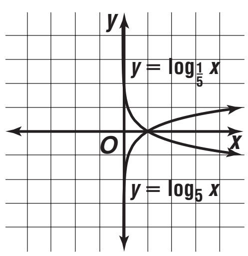


$y = 3^x$ and $\log_3 x$ are similar in that they are both continuous, one-to-one, increasing and inverses.
$y = 3^x$ and $\log_3 x$ are not similar in that they are inverses. The domain of one is the range of another and the range of one is the domain of the other. $y = 3^x$ has a y-intercept and a horizontal asymptote whereas $y = \log_3 x$ has a x-intercept and a vertical asymptote.

2. Let $b^x = m$, then $\log_b m = x$.

$$\begin{aligned} (b^x)^p &= m^p \\ b^{xp} &= m^p \\ \log_b b^{xp} &= \log_b m^p \\ xp &= \log_b m^p \\ p \log_b m &= \log_b m^p \end{aligned}$$

3.



$\text{Log}_5\, x$ is an increasing function and $\log_{\frac{1}{5}} x$ is a decreasing function.

4. Sean is correct. The product property states that $\log_b mn = \log_b m + \log_b n$.

5. In half-life applications $r = -\frac{1}{2}$. So, $(1 + r)$ becomes $\left(1 - \frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)$. Thus, the formula $N = N_0(1 + r)^t$ becomes $N = N_0\left(\frac{1}{2}\right)^t$.

6. $9^{\frac{3}{2}} = 27$

7. $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = 5$

8. $\log_7 y = -6$

9. $\log_8 \frac{1}{4} = -\frac{2}{3}$

10. $$\begin{aligned} \log_2 \tfrac{1}{16} &= x \\ 2^x &= \tfrac{1}{16} \\ 2^x &= 2^{-4} \\ x &= -4 \end{aligned}$$

11. $$\begin{aligned} \log_{10} 0.01 &= x \\ 10^x &= 0.01 \\ 10^x &= 10^{-2} \\ x &= -2 \end{aligned}$$

12. $$\begin{aligned} \log_7 \tfrac{1}{343} &= x \\ 7^x &= \tfrac{1}{343} \\ 7^x &= 7^{-3} \\ x &= -3 \end{aligned}$$

13. $$\begin{aligned} \log_2 x &= 5 \\ 2^5 &= x \\ 32 &= x \end{aligned}$$

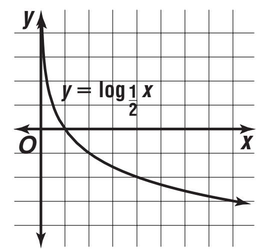
14. $$\begin{aligned} \log_7 n &= \tfrac{2}{3} \log_7 8 \\ \log_7 n &= \log_7 8^{\frac{2}{3}} \\ n &= 8^{\frac{2}{3}} \\ n &= 4 \end{aligned}$$

15. $$\begin{aligned} \log_6 (4x + 4) &= \log_6 64 \\ 4x + 4 &= 64 \\ x &= 15 \end{aligned}$$

16. $$\begin{aligned} 2 \log_6 4 - \tfrac{1}{4} \log_6 16 &= \log_6 x \\ \log_6 4^2 - \log_6 16^{\frac{1}{2}} &= \log_6 x \\ \log_6 \frac{4^2}{16^{\frac{1}{4}}} &= \log_6 x \\ \frac{4^2}{16^{\frac{1}{4}}} &= x \\ x &= 8 \end{aligned}$$

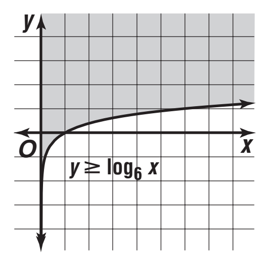
17.

x	y
1	0
2	−1
4	−2



18.

x	y
1	0
6	1



19. $$\begin{aligned} 16 &= \frac{t}{3.3 \log_4 1024} \\ t &= 16(33 \log_4 1024) \\ t &= 16(33 \cdot 5) \\ t &= 264 \text{ h} \end{aligned}$$

$$\begin{aligned} \log_4 1024 &= x \\ 4^x &= 1024 \\ 2^{2x} &= 2^{10} \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Pages 723–725 Exercises

20. $27^{\frac{1}{3}} = 3$

21. $16^{\frac{1}{2}} = 4$

22. $7^{-4} = \frac{1}{2401}$

23. $4^{\frac{5}{2}} = 32$

24. $e^x = 65.98$

25. $\left(\sqrt{6}\right)^4 = 36$

26. $\log_{81} 9 = \frac{1}{2}$

27. $\log_{36} 216 = \frac{3}{2}$

28. $\log_{\frac{1}{8}} 512 = -3$

29. $\log_6 \frac{1}{36} = -2$

30. $\log_{16} 1 = 0$ **31.** $\log_x 14.36 = 1.238$

32. $\log_8 64 = x$

$8^x = 64$ $\quad$ $125^x = 5$

$8^x = 8^2$ $\quad$ $(5^3)^x = 5^1$

$x = 2$ $\quad$ $3x = 1$

33. $\log_{125} 5 = x$ $\quad$ $x = \frac{1}{3}$

34. $\log_2 32 = x$

$2^x = 32$ $\quad$ $4^x = 128$

$2^x = 2^5$ $\quad$ $2^{2x} = 2^7$

$x = 5$ $\quad$ $2x = 7$

35. $\log_4 128 = x$ $\quad$ $x = \frac{7}{2}$ or 3.5

36. $\log_9 9^6 = x$

$9^x = 9^6$ $\quad$ $49^x = 343$

$x = 6$ $\quad$ $7^{2x} = 7^3$

$2x = 3$

37. $\log_{49} 343 = x$

$x = \frac{3}{2}$ or 1.5

38. $\log_8 16 = x$

$8^x = 16$ $\quad$ $\left(\sqrt{8}\right)^x = 4096$

$2^{3x} = 2^4$ $\quad$ $8^{\frac{x}{2}} = 8^4$

$3x = 4$ $\quad$ $\frac{x}{2} = 4$

$x = \frac{4}{3}$ $\quad$ $x = 8$

39. $\log_{\sqrt{8}} 4096 = x$

40. $10^{4 \log_{10} 2} = x$

$10^{\log_{10} 2^4} = x$ $\quad$ **41.** $\log_x 49 = 2$

$2^4 = x$ $\quad$ $x^2 = 49$

$16 = x$ $\quad$ $x = 7$

42. $\log_3 3x = \log_3 36$

$3x = 36$

$x = 12$

43. $\log_6 x + \log_6 9 = \log_6 54$

$\log_6 9x = \log_6 54$

$9x = 54$

$x = 6$

44. $\log_8 48 - \log_8 w = \log_8 6$

$\log_8 \frac{48}{w} = \log_8 6$

$\frac{48}{w} = 6$

$6w = 48$

$w = 8$ $\quad$ $x = 3$

45. $\log_6 216 = x$ $\quad$ **46.** $\log_5 0.04 = x$

$6^x = 216$ $\quad$ $5^x = 0.04$

$6^x = 6^3$ $\quad$ $5^x = 5^{-2}$

$x = -2$

47. $\log_{10} \sqrt[3]{10} = x$

$10^x = \sqrt[3]{10}$

$10^x = 10^{\frac{1}{3}}$

$x = \frac{1}{3}$

48. $\log_{12} x = \frac{1}{2} \log_{12} 9 + \frac{1}{3} \log_{12} 27$

$\log_{12} x = \log_{12} 9^{\frac{1}{2}} + \log_{12} 27^{\frac{1}{3}}$

$\log_{12} x = \log_{12} 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$

$x = 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$

$x = 3 \cdot 3$

$x = 9$

49. $\log_5 (x + 4) + \log_5 8 = \log_5 64$

$\log_5 (x + 4)(8) = \log_5 64$

$(x + 4)(8) = 64$

$x + 4 = 8$

$x = 4$

50. $\log_4 (x - 3) + \log_4 (x + 3) = 2$

$\log_4 (x - 3)(x + 3) = 2$

$4^2 = (x - 3)(x + 3)$

$16 = x^2 - 9$

$25 = x^2$

$5 = x$

51. $\frac{1}{2}(\log_7 x + \log_7 8) = \log_7 16$

$\frac{1}{2}(\log_7 8x) = \log_7 16$

$\log_7 (8x)^{\frac{1}{2}} = \log_7 16$

$(8x)^{\frac{1}{2}} = 16$

$8x = 256$

$x = 32$

52. $2 \log_5 (x - 2) = \log_5 36$

$\log_5 (x - 2)^2 = \log_5 36$

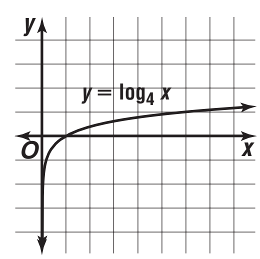
$(x - 2)^2 = 36$

$x - 2 = 6$

$x = 8$

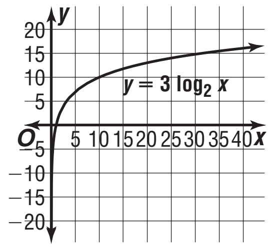
53.

x	y
1	0
2	$\frac{1}{2}$
4	1



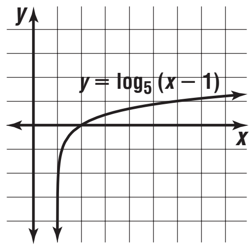
54.

x	y
1	0
2	3
4	6



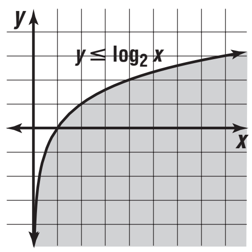
55.

x	y
2	0
6	1



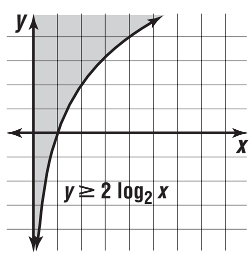
56.

x	y
1	0
2	1
4	2



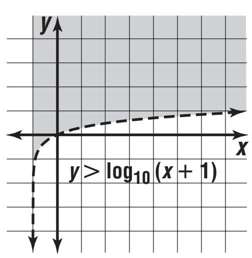
57.

x	y
1	0
2	2
4	4



58.

x	y
0	0
9	1



59. Use $N = N_0(1 + r)^t$; $r = 1$ since the rate of growth is 100% every t time periods.

$64{,}000 = 1000\,(1 + 1)^t$
$64 = 2^t$
$\log_2 2^6 = \log_2 2^t$
$6 = t$
$t \cdot 15 = 90$ min

60. All powers of 1 are 1, so the inverse of $y = 1^x$ is not a function.

61. Let $\log_b m = x$ and $\log_b n = y$.
So, $b^x = m$ and $b^y = n$.

$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$
$\frac{m}{n} = b^{x-y}$
$\log_b \frac{m}{n} = x - y$
$\log_b \frac{m}{n} = \log_b m - \log_b n$

62a. $5000 \geq 2500\left(1 + \frac{r}{4}\right)^{4 \cdot 10}$
$2^7 \geq \left(1 + \frac{r}{4}\right)^{40}$

62b. $2 = \left(1 + \frac{r}{4}\right)^{40}$
$2^{\frac{1}{40}} = \left[\left(1 + \frac{r}{4}\right)^{40}\right]^{\frac{1}{40}}$
$1.0175 \approx 1 + \frac{r}{4}$
$0.0699 \approx r$
6.99%

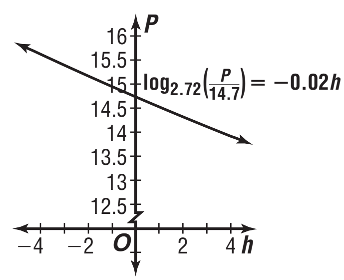
62c. $2 = \left(1 + \frac{r}{4}\right)^{28}$
$2^{\frac{1}{28}} = 1 + \frac{r}{4}$
$1.0251 \approx 1 + \frac{r}{4}$
$0.1004 \approx r$
10.04%

63a. $n = \log_2 \frac{1}{\frac{1}{4}}$
$n = \log_2 4$
$2^n = 4$
$n = 2$

63b. $3 = \log_2 \frac{1}{p}$
$2^3 = \frac{1}{p}$
$8 = p^{-1}$
$\frac{1}{8} = p$
less light; $\frac{1}{8}$

64. Let $y = \log_a x$, so $x = a^y$.
$x = a^y$
$\log_b x = \log_b a^y$
$\log_b x = y \log_b a$
$y = \frac{\log_b x}{\log_b a}$
$\log_a x = \frac{\log_b x}{\log_b a}$

65a.



65b. $\log_{2.72} \frac{P}{14.7} = -0.02(1)$
$\frac{P}{14.7} = 2.72^{-0.02}$
$P = 14.7\,(272^{-0.02})$
≈ 14.4 psi

65c. $\log_{2.72} \frac{P}{14.7} = -0.02(-6.8)$
$P = 14.7(2.72^{0.136})$
≈ 16.84 psi

66. $6.8 = 38\left(\frac{1}{2}\right)^t$
$\frac{6.8}{38} = \left(\frac{1}{2}\right)^t$
$\log \frac{6.8}{38} = \log \left(\frac{1}{2}\right)^t$
$\log \frac{6.8}{38} = t \log \frac{1}{2}$
$t = \frac{\log \frac{6.8}{38}}{\log \frac{1}{2}}$
$t = 2.5$
$2.5 \cdot 3.82 = 9.55$
about 9 days

67. 69.6164

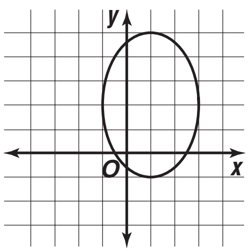
68. $90{,}000 = P\left[\dfrac{1-\left(1+\frac{0.115}{12}\right)^{-12.30}}{\frac{0.115}{12}}\right]$

$P \approx 891.262$

\$891.26

69. ellipse

$$9x^2 - 18x + 4y^2 - 16y - 11 = 0$$
$$9x^2 - 18x + 4y^2 - 16y = 11$$
$$9(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 11 + 9 + 16$$
$$9(x-1)^2 + 4(y-2)^2 = 36$$
$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$$



70. $r = 3, \theta = 2$

$(3 \cos 2t, 3 \sin 2t)$

71. $AB = \sqrt{(-1-(-1))^2 + (-3-3)^2}$
$= \sqrt{0^2 + (-6)^2}$
$= 6$

$BC = \sqrt{(3-(-1))^2 + (0-(-3))^2}$
$= \sqrt{4^2 + 3^2}$
$= 5$

$AC = \sqrt{(3-(-1))^2 + (0-3)^2}$
$= \sqrt{4^2 + 3^2}$
$= 5$

72. $5\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right) \cdot 2\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right)$

$= 5 \cdot 2\left(\cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right) + \boldsymbol{i} \sin\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)\right)$

$= 10\left(\cos \frac{17\pi}{12} + \boldsymbol{i} \sin \frac{17\pi}{12}\right)$

$= 10 \cos \frac{17\pi}{12} + 10\boldsymbol{i} \sin \frac{17\pi}{12}$

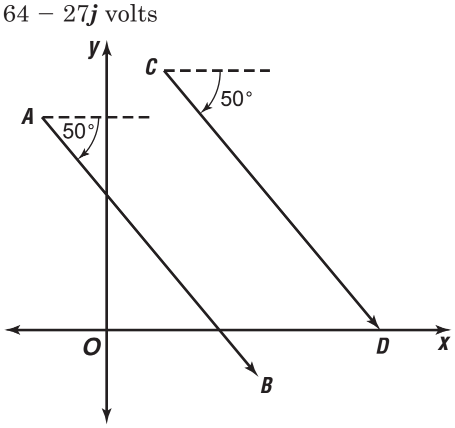
$\approx -2.59 - 9.66\boldsymbol{i}$

$10\left(\cos \frac{17\pi}{12} + \boldsymbol{i} \sin \frac{17\pi}{12}\right), -2.59 - 9.66\boldsymbol{i}$

73. $(3 - 4\boldsymbol{j})(12 + 7\boldsymbol{j}) = 36 + 21\boldsymbol{j} - 48\boldsymbol{j} - 28\boldsymbol{j}^2$
$= 64 - 27\boldsymbol{j}$

$64 - 27\boldsymbol{j}$ volts

74.



Both vectors have the same direction. 50° south of east. Therefore, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

75. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos A = \frac{5}{13}$ | $\cos B = \frac{35}{37}$

$x^2 + 5^2 = 13^2$ | $x^2 + 35^2 = 37^2$

$x^2 = 144$ | $x^2 = 144$

$x = 12$ | $x = 12$

So, $\sin A = \frac{12}{13}$ | So, $\sin B = \frac{12}{37}$

$\cos(A + B) = \frac{5}{13} \cdot \frac{35}{37} - \frac{12}{13} - \frac{12}{37}$
$= \frac{175}{481} - \frac{144}{481}$
$= \frac{31}{481}$

76. $y = A \sin(kt + c) + h$

$A = \frac{90 - 64}{2} = 13$ $\quad h = \frac{90 + 64}{2} = 77$ $\quad \frac{2\pi}{k} = 4$, $k = \frac{\pi}{2}$

$$y = 13 \sin\left(\frac{\pi}{2}t + c\right) + 77$$
$$64 = 13 \sin\left(\frac{\pi}{2}(1) + c\right) + 77$$
$$-13 = 13 \sin\left(\frac{\pi}{2} + c\right)$$
$$-1 = \sin\left(\frac{\pi}{2} + c\right)$$

$\sin^{-1}(-1) - \frac{\pi}{2} = c$

$3.14 \approx c$

So, $y = 13 \sin\left(\frac{\pi}{2}k - 3.14\right) + 77$

77. $c^2 = (6.11)^2 + (5.84)^2 - 2(6.11)(5.84) \cos 105.3$

$c^2 = 37.3321 + 34.1056 - 71.3648 \cos 105.3$

$c^2 \approx 90.2689$

$c \approx 9.5$

$(6.11)^2 \approx (5.84)^2 + (9.5)^2 - 2(5.84)(9.5) \cos A$

$37.3321 \approx 34.1056 + 90.25 - 110.96 \cos A$

$\cos A \approx 0.7843$

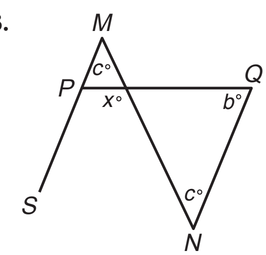
$A \approx 38.34$ or $38° 20'$

$B \approx 180 - (105° 18' + 38° 20')$

$\approx 36° 22'$

$c = 9.5, A = 38° 20', B = 36° 22'$

78.



$m\angle SMN = m\angle QNM$ alternate interior angles

$x° = b° + c°$ Exterior Angle Theorem

The correct choice is E.

11-5 Common Logarithms

Page 730 Check for Understanding

1. log 1 = 0 means $\log_{10} 1 = 0$. So, $10° = 1$.
$\log 10 = 1$ means $\log_{10} 10 = 1$. So, $10^1 = 10$.

2. Write the number in scientific notation. The exponent of the power of 10 is the characteristic.

3. antilog $2.835 = 10^{2.835} = 683.9116$

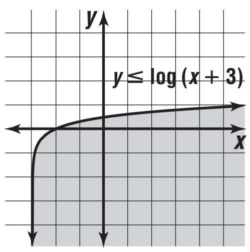
4. $\log 15 = 1.1761$
$\log 5 = 0.6990$
$\log 3 = 0.4771$
$\log 5 + \log 3 = 0.6990 + 0.4771 = 1.1761$

5.
$$\begin{aligned}\log 80{,}000 &= \log (10{,}000 \times 8)\\ &= \log 10^4 + \log 8\\ &= 4 + 0.9031\\ &= 4.9031\end{aligned}$$

6.
$$\begin{aligned}\log 0.003 &= \log (0.001 \times 3)\\ &= \log 10^{-3} + \log 3\\ &= -3 + 0.4771\\ &= -2.5229\end{aligned}$$

7.
$$\begin{aligned}\log 0.0081 &= \log (0.0001 \times 3^4)\\ &= \log 10^{-4} + 4 \log 3\\ &= -4 + 4(0.4771)\\ &= -2.0915\end{aligned}$$

8. 2.6274 **9.** 74,816.95

10.


11.
$$\begin{aligned}\log_{12} 18 &= \frac{\log 18}{\log 12}\\ &\approx 1.1632\end{aligned}$$

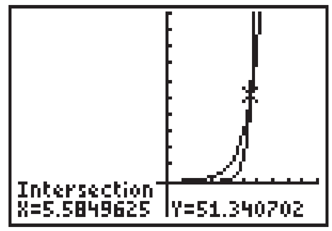
12.
$$\begin{aligned}\log_8 15 &= \frac{\log 15}{\log 8}\\ &\approx 1.3023\end{aligned}$$

13.
$$\begin{aligned}2.2^{x-5} &= 9.32\\ (x - 5) \log 2.2 &= \log 9.32\\ (x - 5) &= \frac{\log 9.32}{\log 2.2}\\ x &\approx 7.83\end{aligned}$$

14.
$$\begin{aligned}6^{x-2} &= 4^x\\ (x - 2) \log 6 &= x \log 4\\ x \log 6 - 2 \log 6 &= x \log 4\\ -2 \log 6 &= x \log 4 - x \log 6\\ -2 \log 6 &= x (\log 4 - \log 6)\\ \frac{-2 \log 6}{\log 4 - \log 6} &= x\\ 8.84 &\approx x\end{aligned}$$

15.
$$\begin{aligned}4.3^x &< 76.2\\ x \log 4.3 &< \log 76.2\\ x &< \frac{\log 76.2}{\log 4.3}\\ x &< 2.97\end{aligned}$$

16.
$$\begin{aligned}3^{x-3} &\geq 2 \sqrt[4]{4^{x-1}}\\ 3^{x-3} &\geq 2\left(4^{\frac{x-1}{4}}\right)\\ (x - 3) \log 3 &\geq \log 2 + \frac{x-1}{4} \log 4\\ (4x - 12)\log 3 &\geq 4 \log 2 + (x - 1) \log 4\\ 4x \log 3 - 12 \log 3 &\geq 4 \log 2 + x \log 4 - \log 4\\ 4x \log 3 - x \log 4 &\geq 4 \log 2 - \log 4 + 12 \log 3\\ x(4 \log 3 - \log 4) &\geq 4 \log 2 - \log 4 + 12 \log 3\\ x &\geq \frac{4 \log 2 - \log 4 + 12 \log 3}{4 \log 3 - \log 4}\\ x &\geq 4.84\end{aligned}$$

17.


[10, 10] scl:1 by [−20, 100] scl:10
5.5850

18a.
$$\begin{aligned}R &= \log \left(\frac{200}{1.6}\right) + 4.2\\ &= 6.3\end{aligned}$$

18b. 10 times; According to the definition of logarithms, R in the equation $R = \log \left(\frac{a}{T}\right) + B$ is an exponent of the base of the logarithm, 10. 10^5 is ten times greater than 10^4.

Pages 730–732 Exercises

19.
$$\begin{aligned}\log 4000{,}000 &= \log (100{,}000 \times 4)\\ &= \log 100{,}000 + \log 4\\ &= 5 + 0.6021\\ &= 5.6021\end{aligned}$$

20.
$$\begin{aligned}\log 0.00009 &= \log (0.00001 \times 9)\\ &= \log 0.00001 + \log 9\\ &= -5 + 0.9542\\ &= -4.0458\end{aligned}$$

21.
$$\begin{aligned}\log 1.2 &= \log (0.1 \times 12)\\ &= \log 0.1 + \log 12\\ &= -1 + 1.0792\\ &= 0.0792\end{aligned}$$

22.
$$\begin{aligned}\log 0.06 &= \log \left(0.01 \times \frac{12}{2}\right)\\ &= \log 0.01 + \log \frac{12}{4^{\frac{1}{2}}}\\ &= \log 0.01 + \log 12 - \frac{1}{2} \log 4\\ &= -2 + 1.0792 - \frac{1}{2}(0.6021)\\ &= -1.2218\end{aligned}$$

23.
$$\begin{aligned}\log 36 &= \log (4 \times 9)\\ &= \log 4 + \log 9\\ &= 0.6021 + 0.9542\\ &= 1.5563\end{aligned}$$

24.
$$\begin{aligned}\log 108{,}000 &= \log (1000 \times 12 \times 9)\\ &= \log 1000 + \log 12 + \log 9\\ &= 3 + 1.0792 + 0.9542\\ &= 5.0334\end{aligned}$$

25. $\log 0.0048 = \log(0.0001 \times 12 \times 4)$
$= \log 0.0001 + \log 12 + \log 4$
$= -4 + 1.0792 + 0.6021$
$= -2.3188$

26. $\log 4.096 = \log(0.001 + 4^6)$
$= \log 0.001 + 6 \log 4$
$= -3 + 6(0.6021)$
$= 0.6124$

27. $\log 1800 = \log\left(100 \times 9 \times 4^{\frac{1}{2}}\right)$
$= \log 100 + \log 9 + \frac{1}{2} \log 4$
$= 2 + 0.9542 + \frac{1}{2}(0.6021)$
$= 3.2553$

28. 1.9921

29. 2.9515

30. 0.871

31. 2.001

32. 3.2769

33. 2.1745

34. $\log_2 8 = \frac{\log 8}{\log 2}$
$= 3$

35. $\log_5 625 = \frac{\log 625}{\log 5}$
$= 4$

36. $\log_6 24 = \frac{\log 24}{\log 6}$
≈ 1.7737

37. $\log_7 4 = \frac{\log 4}{\log 7}$
≈ 0.7124

38. $\log_{6.5} 0.0675 = \frac{\log 0.0675}{\log 0.5}$
≈ 3.8890

39. $\log_{\frac{1}{2}} 15 = \frac{\log 15}{\log \frac{1}{2}}$

40. $2^x = 95$
$x \log 2 = \log 95$
$x = \frac{\log 95}{\log 2}$
$x \approx 6.5699$

41. $5x = 4^{x+3}$
$x \log 5 = (x + 3) \log 4$
$x \log 5 = x \log 4 + 3 \log 4$
$x(\log 5 - \log 4) = 3 \log 4$
$x = \frac{3 \log 4}{\log 5 - \log 4}$
$x \approx 18.6377$

42. $\frac{1}{3} \log x = \log 8$
$x^{\frac{1}{3}} = 8$
$x \approx 512$

43. $0.16^{4+3x} = 0.3^{8-x}$
$(4+3x) \log 0.16 = (8 - x) \log 0.3$
$4 \log 0.16 + 3x \log 0.16 = 8 \log 0.3 - x \log 0.3$
$3x \log 0.16 + x \log 0.3 = 8 \log 0.3 - 4 \log 0.16$
$x(3 \log 0.16 + \log 0.3) = 8 \log 0.3 - 4 \log 0.16$
$x = \frac{8 \log 0.3 - 4 \log 0.16}{3 \log 0.16 + \log 0.3}$
$x \approx 0.3434$

44. $4 \log(x + 3) = 9$
$\log(x + 3) = \frac{9}{4}$
$(x + 3) = \text{antilog } \frac{9}{4}$
$x = \text{antilog } \frac{9}{4} - 3$
$x \approx 174.8297$

45. $0.25 = \log 16^x$
$0.25 = x \log 16$
$x = \frac{0.25}{\log 16}$
$x \approx 0.2076$

46. $3^{x-1} \leq 2^{x-7}$
$(x - 1) \log 3 \leq (x - 7) \log 2$
$x \log 3 - \log 3 \leq x \log 2 - 7 \log 2$
$x \log 3 - x \log 2 \leq \log 3 - 7 \log 2$
$x(\log 3 - \log 2) \leq \log 3 - 7 \log 2$
$x \leq \frac{\log 3 - 7 \log 2}{\log 3 - \log 2}$
$x = -9.2571$

47. $\log_x 6 > 1$
$\frac{\log 6}{\log x} > 1$
$\log 6 > \log x$
$6 > x$
When $x = 1$, $\log 1 = 0$, which means $\frac{\log 6}{\log x}$ is undefined. When $x < 1$, $\frac{\log 6}{\log x}$ is negative, which is not greater than 1. So, x must also be greater than 1. Therefore, $1 < x < 6$.

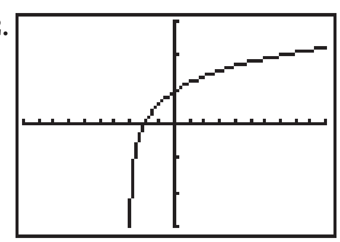
48. $4^{2x-5} \leq 3^{x-3}$
$(2x - 5) \log 4 \leq (x - 3) \log 3$
$2x \log 4 - 5 \log 4 \leq x \log 3 - 3 \log 3$
$2x \log 4 - x \log 3 \leq 5 \log 4 - 3 \log 3$
$x(2 \log 4 - \log 3) \leq 5 \log 4 - 3 \log 3$
$x \leq \frac{5 \log 4 - 3 \log 3}{2 \log 4 - \log 3}$
$x \leq 2.1719$

49. $0.5^{2x-4} \leq 0.1^{5-x}$
$(2x - 4) \log 0.5 \leq (5 - x) \log 0.1$
$2x(\log 0.5) - 4 \log 0.5 \leq 5 \log 0.1 - x \log 0.1$
$2x \log 0.5 + x \log 0.1 \leq 5 \log 0.1 + 4 \log 0.5$
$x(2 \log 0.5 + \log 0.1) \leq 5 \log 0.1 + 4 \log 0.5$
$x \geq \frac{5 \log 0.1 + 4 \log 0.5}{2 \log 0.5 + \log 0.1}$
Change inequality sign because (2 log 0.5 + log 0.1) is negative.
$x \geq 3.8725$

50. $\log_2 x = -3$
$x = 2^{-3}$
$x = 0.1250$

51. $x < \frac{\log 52.7}{\log 3}$
$x < 3.6087$

52.



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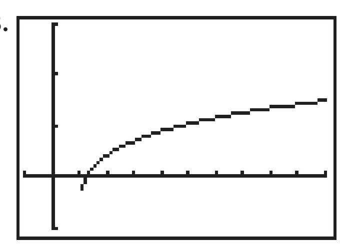
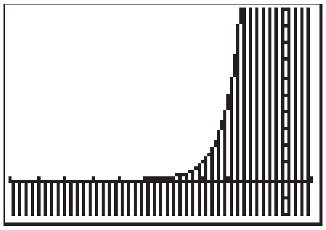
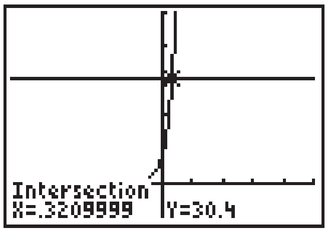
53.



[−1, 10] scl:1 by [−1, 3] scl:1

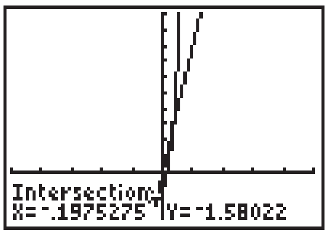
54. 

$[-10, 1]$ scl:1 by $[-2, 10]$ scl:1

55. 

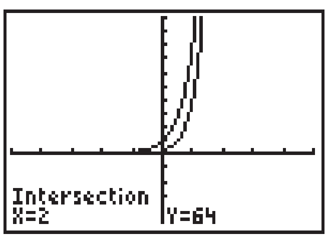
$x \approx 0.3210$

$[-5, 5]$ scl:1 by $[-10, 50]$ scl:10

56. 

$x \approx -0.1975$

$[-5, 5]$ scl:1 by $[-3, 10]$ scl:1

57. 

$x = 2$

$[-5, 5]$ scl:1 by $[-5, 10]$ scl:1

58a. $h = -\frac{100}{9}\log\frac{10.3}{14.7}$
≈ 1.7 mi

58b.
$$\begin{aligned} 4.3 &= -\tfrac{100}{9}\log\tfrac{P}{14.7} \\ -0.3870 &= \log P - \log 14.7 \\ -0.3870 + \log 14.7 &= \log P \\ 0.7803 &\approx \log P \\ 6.03 &\approx P\text{; 6 psi} \end{aligned}$$

59a. $M = 5.3 + 5 + 5\log 0.018$
≈ 1.58

59b.
$$\begin{aligned} 5.3 &= 8.6 + 5 + 5\log P \\ -8.3 &= 5\log P \\ -1.66 &= \log P \\ 0.0219 &\approx P \end{aligned}$$

60a. $q = \left(\frac{1}{2}\right)^{0.8^9}$
$= \left(\frac{1}{2}\right)^{0.1342}$
$= 0.9112$
\$91,116

60b.
$$\begin{aligned} 0.9535 &= \left(\tfrac{1}{2}\right)^{0.8^t} \\ \log 0.9535 &= 0.8^t \log\tfrac{1}{2} \\ \frac{\log 0.9535}{\log\frac{1}{2}} &= 0.8^t \\ \log\left[\frac{\log 0.9535}{\log\frac{1}{2}}\right] &= t\log 0.8 \\ 12.0016 &\approx t \end{aligned}$$
12 years

61. Sample answer: x is between 2 and 3 because 372 is between 100 and 1000, and $\log 100 = 2$ and $\log 1000 = 3$.

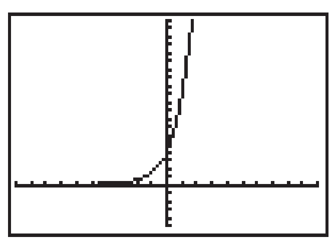
62a. $L = 10\log\frac{1}{1.0 \times 10^{-12}}$
$= 10(\log 1 - \log(1.0 \times 10^{-12}))$
$= 120$ dB

62b.
$$\begin{aligned} 20 &= \log\frac{I}{1.0 \times 10^{-12}} \\ 2 &= \log I - \log(1.0 \times 10^{-12}) \\ 2 &= \log I + 12 \\ -10 &= \log I \end{aligned}$$
$1 \times 10^{-10} = \mathrm{I}$; 1×10^{-10} W/m^2

63. Use $N = N_0\left(\frac{1}{2}\right)^t$.
$N = 630$ micrograms $= 63 \times 10^{-4}$ gram
$N_0 = 1$ milligram $= 1.0 \times 10^{-3}$ gram
$6.3 \times 10^{-4} = (1.0 \times 10^{-3})\left(\frac{1}{2}\right)^t$
$\log\frac{6.3 \times 10^{-4}}{1.0 \times 10^{-3}} = t\log\frac{1}{2}$
$0.6666 \approx t$
$0.6666 \times 5730 \approx 3819$ yr

64. $\log_a y = \log_a P - \log_a q + \log_a r$
$\log_a y = \log_a \frac{p}{q} + \log_a r$
$\log_a y = \log_a \frac{pr}{q}$
$y = \frac{pr}{q}$

65. $\log_x 243 = 5$
$x^5 = 243$
$x = 3$

66. 

increasing from $-\infty$ to ∞

67. $(a^4b^2)^{\frac{1}{3}}c^{\frac{2}{3}} = (a^4)^{\frac{1}{3}}(b^2)^{\frac{1}{3}}(c^2)^{\frac{1}{3}}$
$= a\sqrt[3]{ab^2c^2}$

68. $(5)^2 + (0)^2 + D(5) + E(0) + F = 0$
$5D + F + 25 = 0$
$(1)^2 + (-2)^2 + D(1) + E(-2) + F = 0$
$D - 2E + F + 5 = 0$
$(4)^2 + (-3)^2 + D(4) + E(-3) + F = 0$
$4D - 3E + F + 25 = 0$

$$\begin{array}{rr}
 & 5D + 0E + F + 25 = 0 \\
(-) & \underline{D - 2E + F + 5 = 0} \\
 & 4D + 2E + 20 = 0
\end{array}$$

$$\begin{array}{rr}
 & 4D + 3E + F + 25 = 0 \\
(-) & \underline{D - 2E + F + 5 = 0} \\
 & 3D - E + 20 = 0
\end{array}$$

$$\begin{array}{r}
4D + 2E + 20 = 0 \\
\underline{+2(3D - E + 20 = 0)} \\
10D + 60 = 0 \\
D = -6
\end{array}$$

$$\begin{array}{r}
4(-6) + 2E + 20 = 0 \\
2E - 4 = 0 \\
E = 2
\end{array}$$

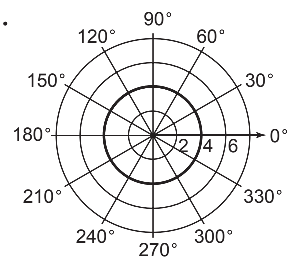
$$\begin{array}{r}
5(-6) + 0(2) + F + 25 = 0 \\
F - 5 = 0 \\
F = 5
\end{array}$$

$$x^2 + y^2 - 6x + 2y + 5 = 0$$
$$(x^2 - 6x + 9) + (y^2 + 2y + 1) = -5 + 9 + 1$$
$$(x - 3)^2 + (y + 1)^2 = 5$$

69. $\left(\frac{6\sqrt{5} - 2\sqrt{5}}{2}, \frac{-18 - 4}{2}\right) = (2\sqrt{5}, -11)$

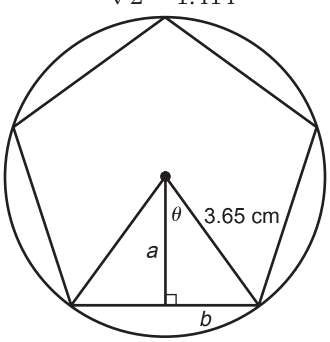
70.
$$\begin{aligned} r &= 6 \\ r^2 &= 36 \\ x^2 + y^2 &= 36 \end{aligned}$$

71.



72. $\overrightarrow{AB} = \langle(6 - 5), (-5 + 6)\rangle$
$= \langle 1, 1\rangle$
$|\overrightarrow{AB}| = \sqrt{(6 - 5)^2 + (-5 + 6)^2}$
$= \sqrt{2} \approx 1.414$

73.



$\theta = 36 \div 10 = 360°$

$\cos 36° = \frac{a}{3.65}$ $\qquad$ $\sin 36° = \frac{b}{3.65}$

$a = 3.65 \cos 36°$ $\qquad$ $b = 3.65 \sin 36°$

≈ 2.9529 $\qquad$ ≈ 2.1454

Use $A = \frac{1}{2}aP$, where $P \approx 10(2.1454) \approx 21.454$.

$A \approx \frac{1}{2}(2.9529)(21.454)$

≈ 31.6758 or $31.68\ \text{cm}^2$

74. $f(x) = x^3 - 2x^2 - 11x + 12$

$f(1) = 1 - 2(1) - 11(1) + 12$ $\qquad$ Test $f(1)$.

$f(1) = 0$ $\qquad$ $(x - 1)$ is a factor.

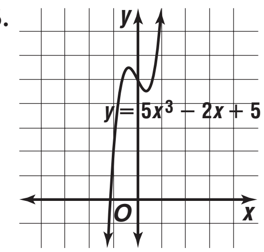
$$\begin{array}{r|rrrr}
1 & 1 & -2 & -11 & 12 \\
 & & 1 & -1 & -12 \\
\hline
 & 1 & -1 & -12 & 0
\end{array}$$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

So, the factors are $(x - 4)(x + 3)(x - 1)$.

75.



Neither; the graph of the function is not symmetric with respect to either the origin or the y-axis.

76. $7 + 5 + 4 + 1 = 17$
17,000,000
The correct choice is D.

11-6 Natural Logarithms

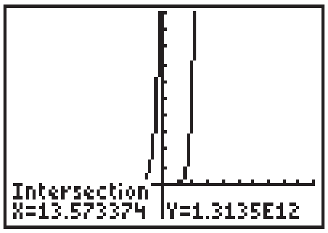
Page 735 Check for Understanding

1. $\ln e = 1$ is the same as $\log_e e = 1$. And $e^1 = e$. So, $\ln e = 1$.
2. The two logarithms have different bases.
$\log 17 \Rightarrow 10^x = 17$ or $x = 1.23$
$\ln 17 \Rightarrow e^x = 17$ or $x = 2.83$
3. $\ln 64 = 4.1589$
$\ln 16 = 2.7726$
$\ln 4 = 1.3863$
$\ln 16 + \ln 4 = 2.7726 + 1.3863 = 4.1589$
4. The two equations represent the same thing, $A = Pe^{rt}$ is a special case of the equation $N = N_0e^{kt}$ and is used primarily for computations involving money.
5. -4.7217
6. 1.1394
7. -15.606
8. 0.4570
9. $\log_5 132 = \frac{\ln 132}{\ln 5}$
≈ 3.0339
10. $\log_3 64 = \frac{\ln 64}{\ln 3}$
≈ 3.7856
11. $$\begin{aligned} 18 &= e^{3x} \\ \ln 18 &= 3x \ln e \\ \frac{\ln 18}{3} &= x \\ 0.9635 &= x \end{aligned}$$

12. $10 = 5e^{5k}$
$2 = e^{5k}$
$\ln 2 = 5k \ln e$
$\frac{\ln 2}{5} = k$
$0.1386 \approx k$

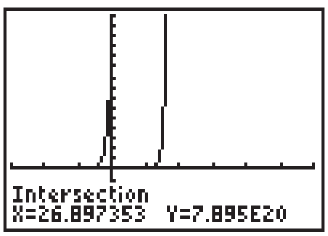
13. $25e^x < 100$
$e^x < 4$
$x \ln e < \ln 4$
$x < 1.3863$

14. $4.5 \geq e^{0.031t}$
$\ln 4.5 \geq 0.031t \ln e$
$\frac{\ln 4.5}{0.031} \geq t$
$48.5186 \geq t$

15. 

$x \approx 13.57$

$[-20, 20]$ scl:2 by $[-4, 20]$ scl:2

16. 

$x > 26.90$

$[-15, 30]$ scl:5 by $[-50, 150]$ scl:10

17a. $p = 760e^{-0.125(3.3)}$
$= 760e^{-0.4125}$
≈ 503.1 torrs

17b. $450 = 760e^{-0.125a}$
$\frac{450}{760} = e^{-0.125a}$
$\ln\left(\frac{450}{760}\right) = -0.125a \ln e$
$\frac{\ln\left(\frac{450}{760}\right)}{-0.125} = a$
$4.1926 \approx$ a; 4.2 km

Pages 736–737 Exercise

18. 5.4931
19. -0.2705
20. 6.8876
21. 0.9657
22. 0
23. 2.2322
24. 10.4395
25. 1.2134
26. 0.0233
27. 0.9966
28. 146.4963
29. 0.2417

30. $\log_{12} 56 = \frac{\ln 56}{\ln 12}$
≈ 1.6199

31. $\log_5 36 = \frac{\ln 36}{\ln 5}$
≈ 2.2266

32. $\log_4 83 = \frac{\ln 83}{\ln 4}$
≈ 3.1875

33. $\log_8 0.512 = \frac{\ln 0.512}{\ln 8}$
≈ -0.3219

34. $\log_6 323 = \frac{\ln 303}{1.6}$
≈ 3.2246

35. $\log_5 \sqrt{288} = \frac{\ln \sqrt{288}}{\ln 5}$
≈ 1.7593

36. $6^x = 72$
$x \ln 6 = \ln 72$
$x = \frac{\ln 72}{\ln 6}$
≈ 2.3869

37. $2^x = 27$
$x \ln 2 = \ln 27$
$x = \frac{\ln 27}{\ln 2}$
≈ 4.7549

38. $9^{x-4} = 7.13$
$(x - 4) \ln 9 = \ln 7.13$
$x \ln 9 - 4 \ln 9 = \ln 7.13$
$x \ln 9 = \ln 7.13 + 4 \ln 9$
$x = \frac{\ln 7.13 + 4 \ln 9}{\ln 9}$
$x \approx 4.8940$

39. $3x = 3\sqrt{2}$
$x \ln 3 = \ln 3 + \ln \sqrt{2}$
$x = \frac{\ln 3 + \ln \sqrt{2}}{\ln 3}$
$x \approx 1.3155$

40. $25e^x = 1000$
$e^x = 40$
$x \ln e = \ln 40$
$x \approx 3.6889$

41. $60.3 < e^{0.1t}$
$\ln 60.3 < 0.1t \ln e$
$\frac{\ln 60.3}{0.1} < t$
$40.9933 < t$

42. $6.2e^{0.64t} = 3e^{t+1}$
$\ln 6.2 + 0.64t \ln e = \ln 3 + (t + 1) \ln e$
$\ln 6.2 - \ln 3 - 1 = 0.36t$
$\frac{\ln 6.2 - \ln 3 - 1}{0.36} = t$
$0.7613 \approx t$

43. $22 = 44\,(1 - e^{2x})$
$\frac{1}{2} - 1 = -e^{2x}$
$\frac{1}{2} = e^{2x}$
$\ln \frac{1}{2} = 2x \ln e$
$\frac{\ln \frac{1}{2}}{2} = x$
$-0.3466 \approx x$

44. $25 < e^{0.075y}$
$\ln 25 < 0.075y \ln e$
$\frac{\ln 25}{0.075} < y$
$y > 42.9183$

45. $5^x \le 7\sqrt{6}$

$x \ln 5 \le \ln 7 + \ln \sqrt{6}$

$x \le \frac{\ln 7 + \ln \sqrt{6}}{\ln 5}$

$x \le 1.7657$

46. $12^{x-4} > 4^x$

$(x - 4) \ln 12 > x \ln 4$

$x \ln 12 - 4 \ln 12 > x \ln 4$

$x \ln 12 - x \ln 4 > 4 \ln 12$

$x(\ln 12 - \ln 4) > 4 \ln 12$

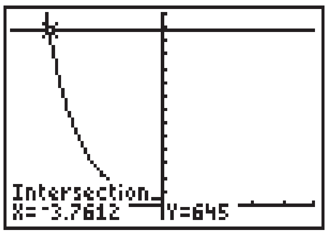
$x > \frac{4 \ln 12}{\ln 12 - \ln 4}$

$x > 9.0474$

47. $x^{\frac{2}{3}} \ge 27.6$

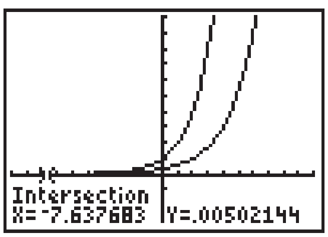
$x \ge (27.6)^{\frac{3}{2}}$

$x \ge 144.9985$

48. 

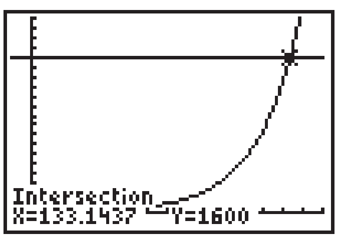
$x \approx -3.76$

[−5, 5] scl:1 by [−50, 700] scl:50

49. 

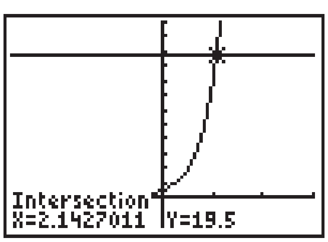
$x \approx -7.64$

[70, 10] scl:1 by [−3, 10] scl:1

50. 

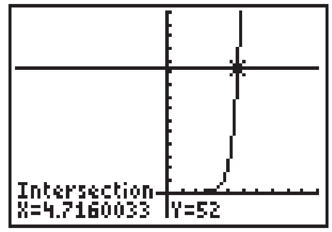
$t < 133.14$

[−10, 150] scl:10 by [−100, 2000] scl:100

51. 

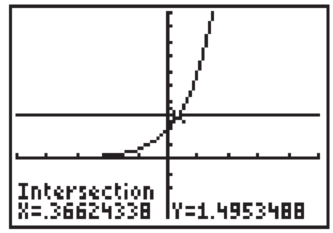
$x \approx 2.14$

[−6, 6] scl:1 by [−4, 24] scl:2

52. 

$x \ge 4.72$

[−10, 10] scl:1 by [−10, 75] scl:5

53. 

$x \ge 0.37$

[−5, 5] scl:1 by [−2, 5] scl: 0.5

54. $0.6 = 1e^{\frac{t}{-20,000(4\times10^{-11})}}$

$\ln 0.6 = \frac{t}{-20,000(4 \times 10^{-11})} \ln e$

$-20,000(4 \times 10^{-11}) \ln 0.6 = t$

$4.09 \times 10^{-7} \approx t$

4.09×10^{-7} s

55. $2.8 = 9\left(\frac{1}{2}\right)^t$

$\ln \frac{2.8}{9} = t \ln \frac{1}{2}$

$\frac{\ln \frac{2.8}{9}}{\ln \frac{1}{2}} = t$

$1.6845 \approx t$

$1.6845 \times 8 \times 24 = 323.4236$

324 h

56a. $\ln |180 - 72| = -k(0) + c$

$4.6821 \approx c$

56b. $\ln |150 - 72| = -k(2) + 4.6821$

$\frac{\ln 78 - 4.6821}{-2} = k$

$0.1627 \approx k$

56c. $\ln |100 - 72| = -(0.1622)t + 4.6821$

$\frac{\ln 28 - 4.6821}{-0.1627} = t$

$8.3 \approx t \qquad 8.3 - 2 = 6.3$

about 6.3 min

57. $e^{-2x} - 4e^{-x} + 3 = 0$

$(e^{-x} - 3)(e^{-x} - 1) = 0$

$e^{-x} - 3 = 0$	$e^{-x} - 1 = 0$
$e^{-x} = 3$	$e^{-x} = 1$
$-x \ln e = \ln 3$	$-x \ln e = \ln 1$
$x \approx -1.0986$	$x = 0$

0 or −1.0986

58a. $2 = e^{0.063t}$

$\ln 2 = 0.063t \ln e$

$\frac{\ln 2}{0.063} = t$

$11.0023 \approx t$

about 11 years

58b. See students' work.

59. $1800 = -5000 \ln r$

$\frac{1800}{-5000} = \ln r$

$\text{antiln}\left(\frac{-1800}{5000}\right) = r$

$0.6977 \approx r$; about 70%

60a. $\frac{1}{2} = 1e^{k(1622)}$

$\ln \frac{1}{2} = 1622\, k \ln e$

$\frac{\ln \frac{1}{2}}{1622} = k$

$-0.000427 \approx k$

60b. $1.7 = 23e^{(-0.000427)(t)}$

$\ln \frac{1.7}{2.3} = -0.000427t \ln e$

$\frac{\ln \frac{1.7}{2.3}}{-0.000427} = t$

$707.9177 \approx t$

about 708 yr

61. y is a logarithmic function of x. The pattern in the table can be determined by $3^y = x$ which can be expressed as $\log_3 x = y$.

62. 1.2844

63. $16^{\frac{3}{4}} = 8$

64. $x^2 = y + 4$

$x^2 + 4y^2 = 8$

$(y + 4) + 4y^2 = 8$

$4y^2 + y - 4 = 0$

$y = \frac{-1 \pm \sqrt{1 - 4(4)(-4)}}{8}$

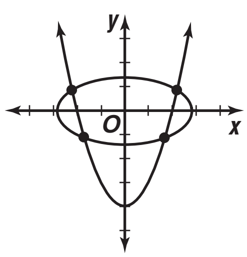
$y = \frac{-1 \pm \sqrt{65}}{8}$

$y \approx 0.9, -1.1$

$x^2 \approx 0.9 + 4$ $\qquad$ $x^2 \approx -11 + 4$

$x \approx \pm \sqrt{4.9}$ $\qquad$ $x \approx \pm\sqrt{2.9}$

$x \approx -2.2, 2.2$ $\qquad$ $x \approx -1.7, 1.7$



65. $\frac{52.4 \text{ N}}{\text{m}^2} \cdot 146 \text{ cm}^3 \cdot \frac{\text{m}^3}{100^3 \text{ cm}^3} = c$

$0.00765 \approx \text{c}; 0.00765 \text{ N} \cdot \text{m}$

66. $x = 0.25 \cos \pi$ $\qquad$ $y = 0.25 \sin \pi$

$= -0.25$ $\qquad$ $= 0$

$(-0.25, 0)$

67. $\vec{a} = \langle 1, -2\rangle + 3\langle 4, 3\rangle$

$= \langle 1, -2\rangle + \langle 12, 9\rangle$

$= \langle 13, 7\rangle$

68. $2x - 5y + 3 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{2^2 + (-5)^2} = -\sqrt{29}$

$-\frac{2x}{\sqrt{29}} + \frac{5y}{\sqrt{29}} - \frac{3}{29} = 0$

$-\frac{2\sqrt{29}}{29}x + \frac{5\sqrt{29}}{29}y - \frac{3\sqrt{29}}{29} = 0$

$p = \frac{3\sqrt{29}}{29} \approx 0.56$ units

$\sin \phi = \frac{5}{\sqrt{29}}$ $\qquad$ $\cos \phi = -\frac{2}{\sqrt{29}}$

$\tan \phi = \frac{\frac{5\sqrt{29}}{29}}{-\frac{2\sqrt{29}}{29}}$

$\tan \phi = -\frac{5}{2}$

$\phi = 112°$

$-\frac{2\sqrt{29}}{29}x + \frac{5\sqrt{29}}{29}y - \frac{3\sqrt{29}}{29} = 0; \frac{3\sqrt{29}}{29} \approx 0.56; 112°$

69. $y = \pm 70 \cos 4\theta$

70. $d = 800 - (10 \cdot 55)$

$= 250$

The correct answer is 250.

11-6B Graphing Calculator Exploration: Natural Logarithms and Area

Pages 738–739

1. 0.69314718

2. 0.6931471806; It is the same value as found in Exercise 1 expressed to 10 decimal places.

3a. The result is the opposite of the result in Exercise 1.

3b. Sample answer: a negative value

4a. 0.69314718

4b. 1.0986123

4c. 1.3862944

4d. 0.6931471806, 1.098612289, 1.386294361

4e. The value for each area is the same as the value of each natural logarithm.

5. −0.5108256238; −0.6931471806; −0.9162907319; These values are equal to the value of ln 0.6, ln 0.5, and ln 0.4.

6. If $k \geq 1$, then the area of the region is equal to $\ln k$. If $0 < k < 1$, then the opposite of the area is equal to $\ln k$.

7. The value of a should be equal to or very close to 1, and the value of b should be very close to e. This prediction is confirmed when you display the actual regression equation.

8. Sample answer: Define $\ln k$ for $k > 0$ to be the area between the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = k$ if $k \geq 1$ and to be the opposite of this area if $0 < k < 1$. Define e to be the value of k for which the area of the region is equal to 1.

11-7 Modeling Real-World Data with Exponential and Logarithmic Functions

Page 744 Check for Understanding

1. Replace N by $4N_0$ in the equation $N = N_0e^{kt}$, where N_0 is the amount invested and k is the interest rate. Then solve for t.

2. The data should be modeled with an exponential function. The points in the scatter plot approach a horizontal asymptote. Exponential functions have horizontal asymptotes, but logarithmic functions do not.

3. $y = 2e^{(\ln 4)x}$ or $y = 2e^{1.3863x}$; $\ln y = \ln 2 + (\ln 4)x$ or $\ln y = 0.6931 + 1.3863x$

4. $t = \frac{\ln 2}{0.0175}$
≈ 39.61 yr

5. $t = \frac{\ln 2}{0.08}$
≈ 8.66 yr

6a. $y = 10.0170(0.9703)^x$

6b. $y = 10.0170(0.9703)^x$
$y = 10.0170(e^{\ln 0.9703})^x$
$y = 10.0170e^{(\ln 0.9703)x}$
$y \approx 10.0170e^{-0.0301x}$

6c. $5 \approx 10.0170e^{-0.0301x}$
$\ln \frac{5}{10.0170} \approx -0.0301x$
$\frac{\ln \frac{5}{10.0170}}{-0.0301} \approx x$
$23.08 \approx x$; 23.08 min

Pages 745–748 Exercises

7. $t = \frac{\ln 2}{0.0225}$
≈ 30.81 yr

8. $t = \frac{\ln 2}{0.05}$
≈ 13.86 yr

9. $t = \frac{\ln 2}{0.07125}$
≈ 9.73

10. exponential; the graph has a horizontal asymptote

11. logarithmic; the graph has a vertical asymptote

12. logarithmic; the graph has a vertical asymptote

13. exponential; the graph has a horizontal asymptote

14a. $y = 4.7818(1.7687)^x$

14b. $y = 4.7818(1.7687)^x$
$y = 4.7818(e^{\ln 1.7687})^x$
$y = 4.7818e^{(\ln 1.7687)x}$
$y = 4.7818e^{0.5702x}$

14c. Use $t = \frac{\ln 2}{k}$; $k = 0.5702$.
$t = \frac{\ln 2}{0.5702}$
≈ 1.215 hr

15a. $y = 1.0091(0.9805)^x$

15b. $y = 1.0091(0.9805)^x$
$y = 1.0091(e^{\ln 0.9805})^x$
$y = 1.0091e^{(\ln 0.9805)x}$
$y = 1.0091e^{-0.0197x}$

15c. $0.415 = 1.0091e^{-0.0197x}$
$\ln \frac{0.415}{1.0091} = -0.0197x$
$\frac{\ln \frac{0.415}{1.0091}}{-0.0197} = x$
$45.10 \approx x$
$45.10 - 10 = 35.10$ min

16a. $y = 2137.5192(1.0534)^x$

16b. $y = 2137.5192(1.0534)^x$
$y = 2137.5192(e^{\ln 1.0534})^x$
$y = 2137.5192e^{(\ln 1.0534)x}$
$y = 2137.5192e^{0.0520x}$

16c. $2631.74 = 2137.52e^{4r}$
$\ln \frac{2631.74}{2137.52} = 4r$
$\ln \frac{\frac{2631.74}{2137.52}}{4} = r$
$0.0520 \approx r$; 5.2%

17. $y = 40 + 14.4270 \ln x$

18a. $y = –826.4217 + 520.4168 \ln x$

18b. The year 1960 would correspond to $x = 0$ and $\ln 0$ is undefined.

19. Take the square root of each side.
$y = cx^2$
$\sqrt{y} = \sqrt{cx^2}$
$\sqrt{y} = \sqrt{c}x$

20a. $1034.34 = 1000(1 + r)^1$
$1.03034 = 1 + r$
$0.03034 \approx r$; 3.034%

20b. $y = 1000.0006(1.0303)^x$

20c. $y = 1000.0006(1.0303)^x$
$y = 1000.0006(e^{\ln 1.0303})^x$
$y = 1000.0006e^{(\ln 1.0303)x}$
$y = 1000.0006e^{0.0299x}$

20d. $1030.34 = 1000e^r$
$\ln \frac{1030.34}{1000} = r$
$0.0299 \approx r$; 2.99%

21a.

x	0	50	100	150	190	200
$\ln y$	1.81	2.07	3.24	3.75	4.25	4.38

21b. $\ln y = 0.0136x + 1.6889$

21c. $\ln y = 0.0136x + 1.6889$
$y = e^{0.0136x+1.6889}$

21d. $y = e^{0.0136(225)+1.6889}$
≈ 115.4572
115.5 persons per square mile

22a. The graph appears to have a horizontal asymptote at $y = 2$, so you must subtract 2 from each y-value before a calculator can perform exponential regression.

22b. $y = 2 + 1.0003(2.5710)^x$

23a. $\ln y$ is a linear function of $\ln x$.
$y = cx^a$
$\ln y = \ln(cx^a)$
$\ln y = \ln c + \ln x^a$
$\ln y = \ln c + a \ln x$

23b. The result of part a indicates that we should take the natural logarithms of both the x- and y-values.

$\ln x$	6.21	6.91	8.52	9.21	9.62
$\ln y$	4.49	4.84	5.65	5.99	6.19

23c. $\ln y = 0.4994 \ln x + 1.3901$

23d. $\ln y = 0.4994 \ln x + 1.3901$

$$e^{\ln y} = e^{0.4994 \ln x + 1.3901}$$
$$y = e^{0.4994 \ln x} \cdot e^{1.3901}$$
$$y = (e^{\ln x})^{0.4994} \cdot 4.0153$$
$$y = 4.0153x^{0.4994}$$

24. $2 = e^{k(85)}$

$$\ln 2 = 85k$$
$$0.0082 \approx k$$
$$12 = e^{0.0082t}$$
$$\ln 12 = 0.0082t$$
$$303 \approx t$$

303.04 min or about 5 h

25. 0.01

26. $\log_5 (7x) = \log_5 (5x + 16)$

$$7x = 5x + 16$$
$$2x = 16$$
$$x = 8$$

27a. $y = x(400 - 20(x - 3))$

$$y = x(460 - 20x)$$
$$y = -20x^2 + 460x$$
$$y - 2645 = 20(x^2 - 23x + 132.25)$$
$$y - 2645 = -20(x - 11.5)$$

vertex at (11.5, 2645), maximum at $x = 11.5$

\$11.50

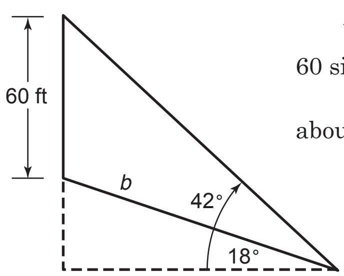
27b. At maximum, $y = 2645$.

\$2645

28. $5\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right) = 5\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}\right)$

$$= \frac{5\sqrt{3}}{2} + \frac{5}{2}\boldsymbol{i}$$

29.



$$\frac{60}{\sin 24°} = \frac{b}{\sin 48°}$$
$$60 \sin 48° = b \sin 24°$$
$$b = 109.625$$

about 109.6 ft

30. $5x^2 - 8x + 12 = 0$

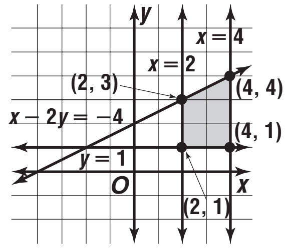
Discriminant: $(-8)^2 - 4(5)(12) = -176$

The discriminant is negative, so there are 2 imaginary roots.

$$x = \frac{8 \pm \sqrt{-176}}{10}$$
$$= \frac{8 \pm 4\boldsymbol{i}\sqrt{11}}{10} \text{ or } \frac{4 \pm 2\boldsymbol{i}\sqrt{11}}{5}$$

31. 4 units left and 8 units down

32.



(2, 1): $f(x) = 2(2) + 8(1) + 10 = 22$

(4, 1): $f(x) = 2(4) + 8(1) + 10 = 26$

(2, 8): $f(x) = 2(2) + 8(3) + 10 = 38$

(4, 4): $f(x) = 2(4) + 8(4) + 10 = 50$

50; 22

33. Circle X contains the regions a, b, d, and e. Circle Z contains the regions d, e, f, and g. Six regions are contained in one or both of circles X and Z.

The correct choice is C.

Chapter 11 Study Guide and Assessment

Page 749 Understanding the Vocabulary

1. common logarithm
2. exponential growth
3. logarithmic function
4. scientific notation
5. mantissa
6. natural logarithm
7. linearizing data
8. exponential function
9. nonlinear regression
10. exponential equation

Pages 750–752 Skills and Concepts

11. $\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = 16$

12. $(64)^{\frac{1}{2}} = 8$

13. $(27)^{\frac{4}{3}} = (3^3)^{\frac{4}{3}} = 3^4 = 81$

14. $(\sqrt[4]{256})^3 = (256)^{\frac{3}{4}} = (4^4)^{\frac{3}{4}} = 4^3 = 64$

15. $3x^2(3x)^{-2} = \frac{3x^2}{(3x)^2} = \frac{3x^2}{9x^2} = \frac{1}{3}$

16. $\left(6a^{\frac{1}{3}}\right)^3 = 6^3\left(a^{\frac{1}{3}}\right)^3 = 216a$

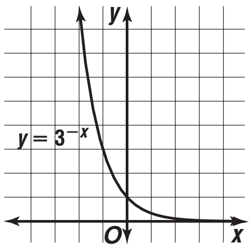
17. $\left(\frac{1}{2}x^4\right)^3 = \left(\frac{1}{2}\right)^3(x^4)^3 = \frac{1}{8}x^{12}$

18. $(w^3)^4 \cdot (4w^2)^2 = w^{12} \cdot 4^2 \cdot w^4 = 16w^{16}$

19. $\left((2a)^{\frac{1}{3}}(a^2b)^{\frac{1}{3}}\right)^3 = \left[(2a)^{\frac{1}{3}}\right]^3 \cdot \left[(a^2b)^{\frac{1}{3}}\right]^2 = (2a)(a^2b) = 2a^3b$

20. $\left(3x^{\frac{1}{2}}y^{\frac{1}{4}}\right)(4x^2y^2) = 12x^{\frac{5}{2}}y^{\frac{9}{4}}$

21.



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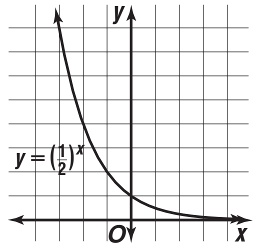

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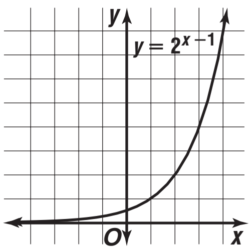

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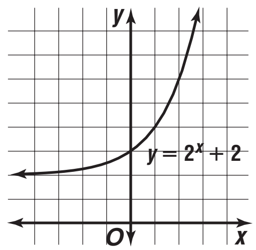

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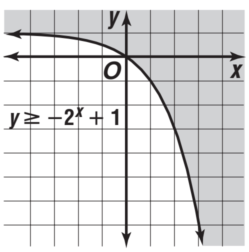
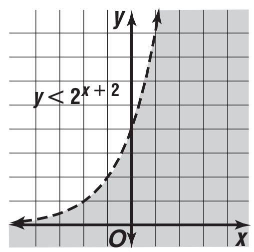


26.



27. $A = 2500e^{0.065(10)}$
≈ 4788.8520; \$4788.85

28. $A = 6000e^{0.0725(10)}$
$\approx 12{,}388.3866$; \$12,388.39

29. $A = 12{,}000e^{0.059(10)}$
$\approx 21{,}647.8610$, \$21,647.86

30. $8^{\frac{2}{3}} = 4$

31. $3^{-4} = \frac{1}{81}$

32. $\log_2 16 = 4$

33. $\log_5 \frac{1}{25} = -2$

34. $2^x = 32$
$2^x = 2^5$
$x = 5$

35. $10^x = 0.001$
$10^x = 10^{-3}$
$x = -3$

36. $4^x = \frac{1}{16}$
$4^x = 4^{-2}$
$x = -2$

37. $2^x = 0.5$
$2^x = 2^{-1}$
$x = -1$

38. $6^x = 216$
$6^x = 6^3$
$x = 3$

39. $9^x = \frac{1}{9}$
$9^x = 9^{-1}$
$x = -1$

40. $4^x = 1024$
$4^x = 4^5$
$x = 5$

41. $8^x = 512$
$8^x = 8^3$
$x = 3$

42. $x^4 = 81$
$x = (81)^{\frac{1}{4}}$
$x = 3$

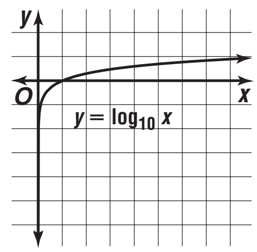
43. $\left(\frac{1}{2}\right)^{-4} = x$
$16 = x$

44. $\log_3 3 + \log_3 x = \log_3 45$
$\log_3 3x = \log_3 45$
$3x = 45$
$x = 15$

45. $2 \log_6 4 - \frac{1}{3} \log_6 8 = \log_6 x$
$\log_6 4^2 - \log_6 8^{\frac{1}{3}} = \log_6 x$
$\log_6 \frac{4^2}{8^{\frac{1}{3}}} = \log_6 x$
$\frac{4^2}{8^{\frac{1}{3}}} = x$
$8 = x$

46. $\log_2 x = \frac{1}{3} \log_6 27$
$\log_2 x = \log_2 27^{\frac{1}{3}}$
$x = 27^{\frac{1}{3}}$
$x = 3$

47.



48. $\log 300{,}000 = \log (100{,}000 \times 3)$
$= \log 100{,}000 + \log 3$
$= 5 + 0.4771$
$= 5.4771$

49. $\log 0.0003 = \log (0.0001 \times 3)$
$= \log 0.0001 + \log 3$
$= -4 + 0.4771$
$= -3.5229$

50. $\log 140 = \log (10 \times 14)$
$= \log 10 + \log 14$
$= 1 + 1.1461$
$= 2.1461$

51. $\log 0.014 = \log (0.001 \times 14)$
$= \log 0.001 + \log 14$
$= -3 + 1.1461$
$= -1.8539$

52. $4x = 6^{x+2}$
$x \log 4 = (x + 2) \log 6$
$x \log^4 = x \log 6 + 2 \log 6$
$x \log 4 - x \log 6 = 2 \log 6$
$x(\log 4 - \log 6) = 2 \log 6$
$x = \frac{2 \log 6}{\log 4 - \log 6}$
$x \approx -8.84$

53. $12^{0.5x} = 8^{0.1x-4}$
$0.5x \log 12 = (0.1x - 4) \log 8$
$0.5x \log 12 = 0.1x \log 8 - 4 \log 8$
$0.5x \log 12 - 0.1x \log 8 = -4 \log 8$
$x(0.5 \log 12 - 0.1 \log 8) = -4 \log 8$
$x = \frac{-4 \log 8}{0.5 \log 12 - 0.1 \log 8}$
$x \approx -8.04$

54. $\left(\frac{1}{4}\right)^{3x} < 6^{x-2}$

$3x \log \frac{1}{4} < (x - 2) \log 6$

$3x \log \frac{1}{4} < x \log 6 - 2 \log 6$

$3x \log \frac{1}{4} - x \log 6 < -2 \log 6$

$x(3 \log \frac{1}{4} - \log 6) < -2 \log 6$

$x > \frac{-2 \log 6}{3 \log \frac{1}{4} - \log 6}$

Change the inequality because 3 log $\frac{1}{4}$ − log 6 is negative.

$x > 0.6$

55. $0 - 1^{2x+8} \geq 7^{x+4}$

$(2x + 8) \log 0.1 \geq (x + 4) \log 7$

$2x \log 0.1 + 8 \log 0.1 \geq x \log 7 + 4 \log 7$

$2x \log 0.1 - x \log 7 \geq 4 \log 7 - 8 \log 0.1$

$x(2 \log 0.1 - \log 7) \geq 4 \log 7 - 8 \log 0.1$

$x \geq \frac{4 \log 7 - 7 \log 0.1}{2 \log 0.1 - \log 7}$

$x \geq -4$

56. $\log (2x + 3) = -\log (3 - x)$

$\log (2x + 3) = \log (3 - x)^{-1}$

$(2x + 3) = (3 - x)^{-1}$

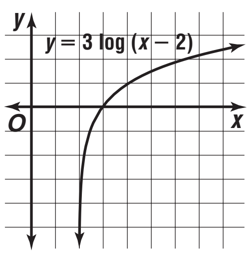
$(2x + 3)(3 - x) = 1$

$2x^2 - 3x - 8 = 0$

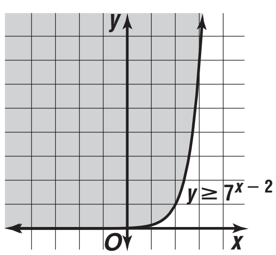
$x = \frac{3 \pm \sqrt{73}}{4}$

$\approx -1.39, 2.89$

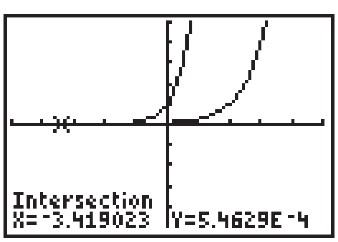
57.



58.



59.



$x = 3.42$

[5, 5] scl:1 by [−5, 5] scl:1

60. $\log_4 15 = \frac{\log 15}{\log 4}$

≈ 1.9534

61. $\log_8 24 = \frac{\log 24}{\log 8}$

≈ 1.5283

62. $\log_4 100 = \frac{\log 100}{\log 9}$

≈ 2.0959

63. $\log_{15} 125 = \frac{\log 125}{\log 15}$

≈ 1.7829

64. $4x = 100$

$x \ln 4 = \ln 100$

$x = \frac{\ln 100}{\ln 4}$

$x \approx 3.3219$

65. $6^{x-2} = 30$

$(x - 2) \ln 6 = \ln 30$

$x - 2 = \frac{\ln 30}{\ln 6}$

$x = \frac{\ln 30}{\ln 6} + 2$

$x \approx 3.8982$

66. $3^{x+1} = 4^{2x}$

$(x + 1) \ln 3 = 2x \ln 4$

$x \ln 3 + \ln 3 = 2x \ln 4$

$x \ln 3 - 2x \ln 4 = -\ln 3$

$x(\ln 3 - 2 \ln 4) = -\ln 3$

$x = \frac{-\ln 3}{\ln 3 - 2 \ln 4}$

$x \approx 0.6563$

67. $9^{4x} = 5^{x-4}$

$4x \ln 9 = (x - 4) \ln 5$

$4x \ln 9 = x \ln 5 - 4 \ln 5$

$4x \ln 9 - x \ln 5 = -4 \ln 5$

$x(4 \ln 9 - \ln 5) = -4 \ln 5$

$x = \frac{-4 \ln 5}{4 \ln 9 - \ln 5}$

$x \approx -0.8967$

68. $24 < e^{2x}$

$\ln 24 < 2x$

$x > \frac{\ln 24}{2}$

$x > 1.5890$

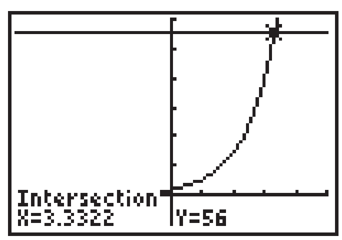
69. $15e^x \geq 200$

$e^x \geq \frac{200}{15}$

$x \geq \ln \frac{200}{15}$

$x \geq 2.5903$

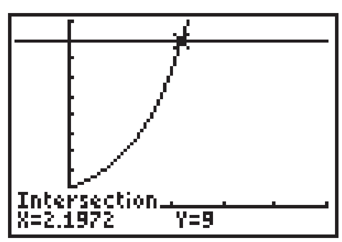
70.



$x \approx 3.333$

[−5, 5] scl:1 by [−10, 60] scl:10

71.



$x < 2.20$

[−1, 5] scl:1 by [−1, 10] scl:1

72. $t = \frac{\ln 2}{0.028}$
≈ 24.76

73. $t = \frac{\ln 2}{0.05125}$
≈ 13.52

74. $18 = \frac{\ln 2}{k}$
$k = \frac{\ln 2}{18}$
≈ 0.0385; 3.85%

Page 753 Applications and Problem solving

75. $0.065 = \left(\frac{1}{2}\right)^t$
$\log 0.65 = t \log \frac{1}{2}$
$\frac{\log 0.65}{\log \frac{1}{2}} = t$
$0.6215 \approx t$
$0.6215 \times 5730 \approx 3561.13$ or 3561 yr.

76a. $\beta = 10 \log \frac{1.15 \times 10^{-10}}{10^{-12}}$
≈ 20.6 20.6 dB

76b. $\beta = 10 \log \frac{9 \times 10^{-9}}{10^{-2}}$
≈ 39.5 39.5 dB

76c. $\beta = 10 \log \frac{8.95 \times 10^{-3}}{10^{-12}}$
≈ 99.5 99.5 dB

77. $200{,}000 = 142{,}000e^{0.014t}$
$\frac{100}{71} = e^{0.014t}$
$\ln \frac{100}{71} = 0.014t$
$\frac{\ln \frac{100}{71}}{0.014} = t$
$24.4 \approx t$
$1990 + 24 = 2014$

78a. $N = 65 - 30e^{-0.20(2)}$
≈ 44.89; 45 words per minute

78b. $N = 65 - 30e^{-0.20(15)}$
$= 63.50$; 64 words per minute

78c. $50 = 65 - 30e^{-0.20t}$
$\frac{1}{2} = e^{-0.20t}$
$\ln \frac{1}{2} = -0.20t$
$\frac{\ln \frac{1}{2}}{-0.20} = t$
$3.47 \approx t$; 3.5 weeks

Page 753 Open-Ended Assessment

1. Sample answer: $(n^4)^{\frac{1}{4}}(4m)^{-1}$
2. Sample answer:
$\log 2 + \log(x + 2) = \frac{1}{2} \log 36$

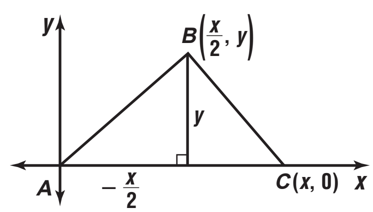
Chapter SAT & ACT Preparation

Page 755 SAT and ACT Practice

1. To find the greatest possible value, the other 3 values must be as small as possible. Since they are distinct positive integers, they must be 1, 2, and 3. The sum of all 4 integers is 4(11) or 44. The sum of the 3 smallest is $1 + 2 + 3$ or 6, so the fourth integer cannot be more than $44 - 6$ or 38. The correct choice is B.

2. Since one root is $\frac{1}{2}$, $x = \frac{1}{2}$, $2x = 1$, and $2x = 0$. Similarly for the root that is $\frac{1}{3}$,
$x = \frac{1}{3}$, $3x = 1$, and $3x - 1 = 0$.
To find the quadratic equation, multiply these two factors and let the product equal zero.
$(2x - 1)(3x - 1) = 0$
$6x^2 - 5x + 1 = 0$
The correct choice is E.

3. The result of dividing T by 6 is 14 less than the correct average.
$\frac{T}{6} =$ correct answer $- 14$
$\frac{T}{6} + 14 =$ correct average
The correct average is the total divided by the number of scores, 5.
correct average $= \frac{T}{5}$
$\frac{T}{6} + 14 = \frac{T}{5}$
The correct choice is E.

4. 

$\tan A = \frac{y}{\frac{x}{2}} = \frac{2y}{x}$
Find the area of $\triangle ABC$.
$A = \frac{1}{2}bh = \frac{1}{2}xy = \frac{xy}{2}$
Simplify the ratio.
$\frac{\text{area of } \triangle ABC}{\tan A} = \frac{\frac{xy}{2}}{\frac{2y}{x}} = \frac{x}{2y}\left(\frac{xy}{2}\right) = \frac{x^2}{4}$
The correct choice is E.

5. $\frac{x}{y} = \frac{10}{2y}$
$2yx = 10y$
$2x = 10$
$x = 5$
The correct choice is B.

6. $C = \pi D$

$\frac{2\pi}{3} = \pi D$

$D = \frac{2}{3}$ and, therefore, $r = \frac{1}{3}$.

Now use this value for the radius to calculate *half* of the area.

$\frac{1}{2}A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{3}\right)^2 = \frac{1}{2}\pi\left(\frac{1}{9}\right) = \frac{\pi}{18}$

The correct choice is A.

7. The average of 8 numbers is 20.

$20 = \frac{\text{sum of eight numbers}}{8}$

sum of eight numbers = 160

The average of 5 of the numbers is 14.

$14 = \frac{\text{sum of five numbers}}{5}$

sum of five numbers = 70

The sum of the other three numbers must be $160 - 70$ or 90. Calculate the average of these three numbers.

$\text{average} = \frac{\text{sum of three numbers}}{3} = \frac{90}{3} = 30$

The correct choice is D.

8. The sum of the angles in a triangle is 180°. Since $\angle B$ is a right angle, it is 90°. So the sum of the other two angles is 90°. Write and solve an equation using the expressions for the two angles.

$2x + 3x = 90$

$5x = 90$

$x = 18$

The question asks for the measure of $\angle A$.

$\angle A = 2x = 2(18) = 36$

The correct choice is C.

9. A is the arithmetic mean of three consecutive positive even integers, so $A = \frac{x + (x + 2) + (x + 4)}{3} = \frac{3x + 6}{3} = x + 2$, where x is a positive even integer. Then A is also a positive even integer. Since A is even, when A is divided by 6, the remainder must also be an even integer. The possible even remainders are 0, 2, and 4.

The correct choice is C.

10. First notice that b must be a prime integer. Next notice that $3b$ is greater than 10. So b could be 5, since $3(5) = 15$. (b cannot be 3.) Check to be sure that 5 fits the rest of the inequality.

$3(5) = 15 > \frac{5}{6}(5) = \frac{25}{6} = 4\frac{1}{6}$

So 5 is one possible answer. You can check to see that 7 and 11 are also valid answers.

The correct answer is 5, 7, or 11.